

Math 106 Section 550 Exam 1 Practice problems

The following problems will help you in preparing for the first exam, **in addition to** the problems assigned from the text for the sections covered by the exam (see the review sheet). There are more problems (perhaps twice as many) here than will appear on the exam.

You should show all of your work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it! You should solve each problem using the methods explored up to this point in class and the text, together with any necessary algebra, trigonometry, or geometry, as appropriate.

1. Determine the following limits:

$$\lim_{x \rightarrow 2} \frac{x^3 - x^2 - 8}{2x^2 + 3x - 14}$$

$$\lim_{x \rightarrow 0} \frac{x \sin x - \cos x}{x \sin x + \cos x}$$

2. Find the vertical and horizontal asymptotes of the function

$$f(x) = \frac{2x + 15}{(x - 3)^3}$$

and determine the limiting values of f as x approaches each side of your vertical asymptotes.

3. Sketch the graph of a function $y = f(x)$ having horizontal asymptotes $y = 3$ and $y = 4$, and vertical asymptotes at $x = -2$ and $x = 1$, and $f(x) \rightarrow \infty$ as $x \rightarrow -2^-$, -2^+ and 1^+ , and $f(x) \rightarrow -\infty$ as $x \rightarrow 1^-$. [There is more than one correct answer to this question!]
4. Show that the function $f(x) = x^4 + 2x^3 + 7x + 5$ has at least *two* distinct roots ($f(c) = 0$) in the interval $[-3, 3]$.

5. Find the discontinuities of the function

$$f(x) = \begin{cases} x, & \text{if } -\infty < x \leq -2 \\ x^2 - 6, & \text{if } -2 \leq x < 1 \\ 3, & \text{if } x = 1 \\ 2x - 7, & \text{if } 1 < x < 4 \\ x^2, & \text{if } 4 \leq x < \infty \end{cases}$$

and determine which of (removable, jump, infinite) each discontinuity is.

6. (a): Find, using (one of) the (limit) definitions of the derivative, the derivative of the indicated function at the indicated point, and find the equation for the tangent line to the graph of the function at the same point:

$$f(x) = x^2 - 3x + 10 \text{ at } x = 2.$$

$$f(x) = \frac{2}{x-3} \text{ at } x = 4.$$

7. Find, using any method, the derivatives of the functions:

$$f(x) = 5x^{-3/2} + \frac{1}{x^2 - x + 3}$$

$$g(x) = x^{1/2}(x^2 + 3x - 3)$$

$$h(x) = \sin^2 x \cos x$$

8. If A and B are functions of x , $A(2) = 1$, $B(2) = -2$, $A'(2) = 3$, and $vB'(2) = 4$, what is the derivative of $\frac{A}{5+B}$ at $x = 2$? What is the derivative of $3xAB$ at $x = 2$?