Name: Solutions

Math 106 Section 550 Exam 2

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it! You should solve each problem using the methods explored up to this point in class and the text, together with any necessary algebra, trigonometry, or geometry, as appropriate.

1. (15 pts.) Find the equation of the line tangent to the graph of the function $y = f(x)$ defined implicitly by the equation
$$y^3 + 2xy^2 + x^2 = 1$$
at the point $(-1, 2)$.

$$3y^2y' + 2(y^2 + x(2yy')) + 2x = 0$$

$$y'(3y^2 + 4xy) + (2y^2 + 2x) = 0$$

$$y' = \frac{-(2y^2+2x)}{(3y^2+4xy)}$$

at $(-1,2)$: $$y' = \frac{-(2(4)+2(-1))}{3(4)+4(-1)(2)} = \frac{-(6)}{12-8}$$

$$= \frac{-6}{4} = -\frac{3}{2}$$

Tangent line: $$\boxed{y-2 = -\frac{3}{2}(x-(-1))}$$

$$y = 2 - \frac{3}{2}x - \frac{3}{2} = -\frac{3}{2}x + \frac{1}{2}$$

2. (15 pts.) Use the tangent line to the graph of the function $h(x) = x^{1/3}$ at $x = 27$ to approximate the value of $5^{2/3} = (25)^{1/3}$

$$h'(x) = \frac{1}{3} x^{-2/3}$$

$$h(27) = 27^{1/3} = 3$$

$$h'(27) = \frac{1}{3} \frac{1}{((27)^{1/3})^2} = \frac{1}{3} \cdot \frac{1}{9} = \frac{1}{27}$$

tangent line:

$$y - 3 = \frac{1}{27}(x - 27)$$

$$y = 3 + \frac{1}{27}(x - 27) = L(x)$$

So $$5^{2/3} = h(25) \approx L(25) = \boxed{3 + \frac{1}{27}(25 - 27)}$$

$$= 3 - \frac{2}{27} = \frac{81-2}{27} = \frac{79}{27}$$

3. (15 pts.) Use logarithmic differentiation to find the derivative of the function

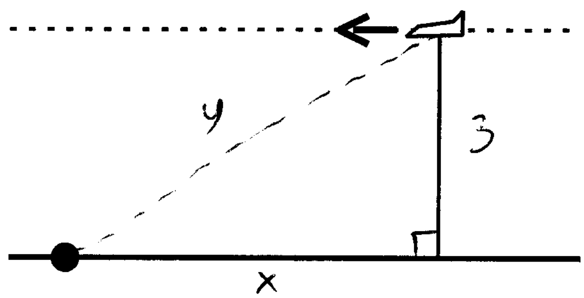
$$g(x) = (x^2 + \tan x)^3 (xe^x - 4)^{\frac{11}{2}}$$

$$\ln(g(x)) = 3\ln(x^2+\tan x) + \frac{11}{2}\ln(xe^x - 4)$$

$$g'(x) = g(x)(\ln(g(x)))' = g(x)\left(3\left(\frac{2x+\sec^2 x}{x^2+\tan x}\right) + \frac{11}{2}\left(\frac{e^x + xe^x}{xe^x - 4}\right)\right)$$

$$= (x^2+\tan x)^3(xe^x-4)^{\frac{11}{2}}\left(3\left(\frac{2x+\sec^2 x}{x^2+\tan x}\right) + \frac{11}{2}\left(\frac{e^x+xe^x}{xe^x-4}\right)\right)$$

4. (20 pts.) A plane flying at a constant altitude of 3 miles is flying directly at a radar station. If the radar station determines that when the distance from the plane to station is 5 miles that the distance from the plane to the station is decreasing at a rate of 600 miles/hour, what is the speed of the plane?



$x^2+9=y^2$

When $y=5$, $\frac{dy}{dt}=-600$,

what is $\frac{dx}{dt}$?

$$2x\frac{dx}{dt}+0=2y\frac{dy}{dt}$$

$$\frac{dx}{dt}=\frac{y}{x}\frac{dy}{dt}$$

$y=5$

$\frac{dy}{dt}=-600$

$x^2+9=5^2=25$

$x^2=25-9=16$

$x=\sqrt{16}=4$

$$\frac{dx}{dt}=\frac{5}{4}(-600)=-750$$

speed $= 750$ miles/hr.

5. (20 pts.) Find the absolute maximum and minimum values of the function

$$g(x) = x^4 - 4x^3 - 2x^2 + 12x - 5$$

on the interval $[-3, 3]$.

$$g'(x) = 4x^3 - 12x^2 - 4x + 12$$
$$= 4(x^3 - 3x^2 - x + 3) = 0$$

Try $x=1$: $1-3-1+3=0$ ✓

$$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2\pm 4}{2} = \frac{-2}{2}, \frac{6}{2}$$

$$g'(x) = 4(x-1)(x^2-2x-3)$$
$$= 4(x-1)(x+1)(x-3)$$
$$= 0 \quad x = 1, -1, 3$$

$g'(x)$ DNE NEVER.

critical pts: $-1, 1, 3$ endpts: $-3, 3$

$f(-3) = 81 + 108 - 18 - 36 - 5 = 189 - 59 = 130$

$f(-1) = 1 + 4 - 2 - 12 - 5 = 5 - 19 = -14$

$f(1) = 1 - 4 - 2 + 12 - 5 = 13 - 11 = 2$

$f(3) = 81 - 108 - 18 + 36 - 5 = 117 - 131 = -14$

Abs max value $= \boxed{130}$ (at $x=-3$)

Abs min value $= \boxed{-14}$ (at $x=-1$ and $x=3$)

6. (15 pts.) The function $g(x) = x^4 - 3x^3 + ax^2 - 6x + 1$ has a point of inflection at $x = 1$. What is the value of a ? (Be sure to verify that your chosen value of a <u>does</u> produce a function with a point of inflection at 1 !)

For p.o.i., <u>need</u> $g''(x) = 0$ or DNE

$g'(x) = 4x^3 - 9x^2 + 2ax - 6$

$g''(x) = 12x^2 - 18x + 2a$ DNE? <u>never</u>.

<u>Want</u> $\cancel{g''(x)}$

$0 = g''(1) = 12 - 18 + 2a = 2a - 6$

so $2a = 6$; $\boxed{a = 3}$

<u>Then</u>: $g(x) = x^4 - 3x^3 + 3x^2 - 6x + 1$

so $g'(x) = 4x^3 - 9x^2 + 6x - 6$

$g''(x) = 12x^2 - 18x + 6 = 6(2x^2 - 3x + 1)$

$= 6(2x-1)(x-1)$

g'' + + 0 - - 0 + +

$\frac{1}{2}$ 1

Concavity <u>does</u> change sign at $x = 1$, so $x = 1$ <u>is</u> a point of inflection.