Math 106 Section 550 Exam 2 Practice problems

The following problems will help you in preparing for the second exam, **in addition to** the problems assigned from the text for the sections covered by the exam (see the review sheet). There are more questions (perhaps twice as many) here than will appear on the exam.

You should show all of your work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it! You should solve each problem using the methods explored up to this point in class and the text, together with any necessary algebra, trigonometry, or geometry, as appropriate.

1. Find the derivatives of the functions
   (a): $f(x) = \sin(x^2 e^{-x})$
   (b): $g(x) = \arctan(x^3)$
   (c): $h(x) = e^{\sin(\ln x)} - e^{\ln(\sin x)}$

2. Suppose that we are given a function $y = g(x)$ with $g(3) = 2$ and $g'(3) = -5$. What is the derivative of $H(x) = (2 + g(4x-1))^{11}$ at $x = 1$?

3. Fnd the equation of the line tangent to the parametric curve $(x(t), y(t)) = (t^2 \ln t, 2t^{3/2} - t^3)$ at $t = 1$.

4. Find $\dfrac{dy}{dx}$ for the function $y = f(x)$ defined implicitly by the equation
$$x^3 + xy - y^3 = 4$$

5. The function $f(x) = x^3 - x^2 + 9x + 7$ is invertible [[Not really part of the problem: why?]]; find the derivative of the inerse $g(x) = f^{-1}(x)$ of f at $x = 29$. (Hint: $f(x) = 29$ for a reasonable (and reasonably small) number...)

6. Use logarithmic differentiation to find the derivative of the function
$$g(x) = \frac{(x^2+3)^{5/3}(x^2+\cos x)^4}{(\ln x)^{2+x)}}$$

7. Two people are on a baseball field, one running from home plate to first base and the other from third to home. At one point in time, the first runner is 30 feet from first base and travelling 8 ft/sec, and the other runner is 40 feet from home plate and travelling 15 feet/sec. How fast is the distance between them changing at this point in time? (Note: the regulation distance between bases is 90 feet. Hint: draw the bases so that the runners are travelling vertically/horizontally....)

8. Use the tangent line to the graph of $f(x) = \sqrt{x}$ at $x = 49$ to approximate the value of $\sqrt{51}$.

9. A company which manufactures construction materials is required by its contract to supply cubical bricks with a weight of 4 pounds, plus or minus .1 pound. If the bricks are made of a material with a density of 32 pounds per cubic foot, use differentials to estimate the maximum error allowed in the length of the sides of the cube.

10. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 2x^2 - 7x + 4$ on the interval $[-2, 2]$.

11. Find the critical points of the function $g(x) = (x^3 - 4)e^x$, and for each, determine if it is a relative max, a relative min, or neither.

12. Show that a function $y = f(x)$ with derivative $f'(x) = e^x \cos^2(x) + x^{212}$ can have <u>at most</u> one root. (Hint: if $A \geq 0, B \geq 0$ and $A + B = 0$, what can you say about A and B?)

13. For the function $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x + 3$, determine where the function is increasing, decreasing, concave up, and concave down, and use this information to provide a rough sketch of the graph of f.