Math 106 Section 550 Exam 2 Practice Solutions

$\boxed{1.}$ (a) $(\sin(x^2e^{-x}))' = \cos(x^2e^{-x})(x^2e^{-x})'$
$= (\cos(x^2e^{-x}))(2xe^{-x} - x^2e^{-x})$

(b) $(\arctan(x^3))' = \frac{1}{1+(x^3)^2}(x^3)' = \frac{3x^2}{1+x^6}$

(c) $(e^{\sin(\ln x)} - e^{\ln(\sin x)})' = e^{\sin(\ln x)}(\sin(\ln x))'$
$- e^{\ln(\sin x)}(\ln(\sin x))'$
$= e^{\sin(\ln x)}(\cos(\ln x))(\ln x)' - e^{\ln(\sin x)}\frac{1}{\sin x}(\sin x)'$
$= e^{\sin(\ln x)}(\cos(\ln x))\frac{1}{x} - e^{\ln(\sin x)}\frac{\cos x}{\sin x}$

$\boxed{2.}$ $g(3)=2$, $g'(3)=-5$
$H'(x) = ((2+g(4x-1))^{11})' = 11(2+g(4x-1))^{10}(2+g(4x-1))'$
$= 11(2+g(4x-1))^{10}(g'(4x-1))(4)$
At $x=1$, $H'(1) = 11(2+g(3))^{10}(g'(3))(4)$
$= 11(2+2)^{10}(-5)(4) = 11\cdot 4^{10}(-5)(4)$.

$\boxed{3.}$ $x = t^2\ln t$, $y = 2t^{3/2} - t^3$
$x' = 2t\ln t + t^2(\frac{1}{t}) = t + 2t\ln t$, $y' = 2\frac{3}{2}t^{1/2} - 3t^2 = 3t^{1/2} - 3t^2$
at $t=1$, $x' = 1 + 2\cdot 1\cdot 0 = 1$, $y' = 3\cdot 1 - 3\cdot 1 = 0$
$dy/dx = (dy/dt)/(dx/dt) = 0/1 = 0$ $\quad x = 1^2\cdot 0 = 0$, $y = 2\cdot 1 - 1 = 1$
tangent line: $(y-1) = 0(x-0) = 0$; $\boxed{y=1}$

4. $x^3+xy-y^3=4$; $3x^2+\left(y+x\frac{dy}{dx}\right)-3y^2\left(\frac{dy}{dx}\right)=0$

$3x^2+y-(3y^2-x)\frac{dy}{dx}=0$ $\boxed{\frac{dy}{dx}=\frac{3x^2+y}{3y^2-x}}$

5. $f(x)=x^3-x^2+9x+7$ is invertible

[$f'(x)=3x^2-2x+9=3(x^2-\frac{2}{3}x+3)=3\left((x-\frac{1}{3})^2+(3-\frac{1}{9})\right)$ is always >0, so Rolle's Thm says f never takes the same value twice, so f is 1-to-1.]

and $f(0)=7$, $f(1)=16$, $f(-1)=-4$, $f(2)=8-4+18+7=\boxed{29}$

so $f^{-1}(29)=2$, and $(f^{-1})'(29)=\frac{1}{f'(f^{-1}(29))}=\frac{1}{f'(2)}$

$=\frac{1}{3(2)^2-2(2)+9}=\frac{1}{12-4+9}=\boxed{\frac{1}{17}}$

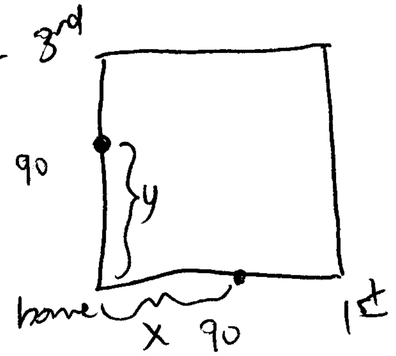
6. $g(x)=\frac{(x^3+3)^{5/3}(x^2+\cos x)^4}{(\ln x)^{2+x}}$

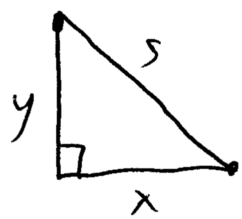
$\ln(g(x))=\frac{5}{3}\ln(x^3+3)+4\ln(x^2+\cos x)-(2+x)\ln(\ln x)$

$g'(x)=g(x)(\ln(g(x)))'=g(x)\left(\frac{5}{3}\frac{2x}{x^3+3}+4\frac{2x-\sin x}{x^2+\cos x}\right.$ $\left.-\left[(1)\ln(\ln x)+(2+x)\frac{1}{\ln x}\left(\frac{1}{x}\right)\right]\right)$

$=\frac{(x^3+3)^{5/3}(x^2+\cos x)^4}{(\ln x)^{2+x}}\left[\frac{5}{3}\frac{2x}{x^3+3}+4\frac{2x-\sin x}{x^2+\cos x}-\left[\ln(\ln x)+\frac{2+x}{x\ln x}\right]\right]$

7.





$s^2 = x^2 + y^2$

What is $\frac{ds}{dt}$ when

$x = 90 - 30 = 60$, $y = 40$

$\frac{dx}{dt} = 8$ $\frac{dy}{dt} = -15$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$s^2 = (60)^2 + (40)^2 = (20)^2(9+4)$$

$s = 20\sqrt{13}$

$$2(20\sqrt{13})\frac{ds}{dt} = 2(60)(8) + 2(40)(-15)$$

$$\boxed{\frac{ds}{dt} = \frac{1}{2(20\sqrt{13})}\left(2(60)(8) - 2(40)(15)\right)}$$

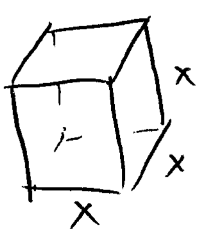
8. $f(x) = \sqrt{x}$ at $x = 49$ $f(49) = \sqrt{49} = 7$

$f'(x) = \frac{1}{2}x^{-1/2}$ $f'(49) = \frac{1}{2}49^{-1/2} = \frac{1}{14}$

$$L(x) = 7 + \frac{1}{14}(x - 49)$$

$$\sqrt{51} = f(51) \approx L(51) = 7 + \frac{1}{14}(51 - 49) = 7 + \frac{2}{14} = \boxed{7 + \frac{1}{7}}$$

9.



$M(x) \approx$ mass $=$ vol $\cdot$ density $= x^3 \cdot 32 = 32x^3 = 4$

$x^3 = \frac{1}{8}$ $x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

$\cancel{dM = 32(3x^2)dx}$

At $x = \frac{1}{2}$,

$.1 = dM = 32(3x^2)\,dx = 32 \cdot 3\left(\frac{1}{2}\right)^2 dx = 24\,dx$, so

$dx = \frac{1}{10 \cdot 24} = \frac{1}{240}$ = max error in x.

$\boxed{10.}$ $f(x) = x^3 - 2x^2 - 7x + 4$ on $[-2,2]$

$f'(x) = 3x^2 - 4x - 7 = 0 \quad x = \dfrac{4 \pm \sqrt{16+84}}{6} = \dfrac{4 \pm 10}{6}$

$= \dfrac{-6}{6}, \dfrac{14}{6} = -1, \cancel{7/3}$

$\uparrow$ not in $[-2,2]$

[$f'(x)$ DNE? never.]

candidates: $x = -2, -1, 2$

$f(-2) = -8 - 8 + 14 + 4 = 2$

$f(-1) = -1 - 2 + 7 + 4 = 8 \longleftarrow$ abs max. value

$f(2) = 8 - 8 - 14 + 4 = -10 \longleftarrow$ abs min. value

$\boxed{11.}$ $g(x) = (x^3 - 4)e^x$

$g'(x) = (3x^2)e^x + (x^3 - 4)e^x$

$= (x^3 + 3x^2 - 4)e^x \quad$ ($=0$ at $x=1$, $(x-1)$ is factor)

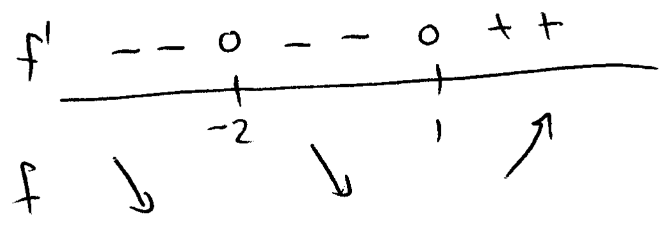
$= (x-1)(x^2 + 4x + 4)e^x$

$= (x-1)(x+2)^2 e^x = 0 \quad$ $x = 1$, $x = -2$

($e^x = 0$ never)

[$g'(x)$ DNE? never.]

critical points: $x = 1, -2$



$x = 1$ is rel min. (by the 1st derivative test.)

$x = -2$ is neither.

$\boxed{12.}$ $f'(x) = e^x\cos^2 x + x^{212} \geq 0$ since e^x, $\cos^2 x$, and x^{212} are always ≥ 0.

$f'(x) = 0 = (e^x\cos^2 x) + (x^{212})$

needs $e^x\cos^2 x = 0$ and $x^{212} = 0$; so $x=0$ but then $e^x\cos^2 x = (1)(1)^2 = 1$. So $f'(x)$ is always $\underline{\underline{> 0}}$,

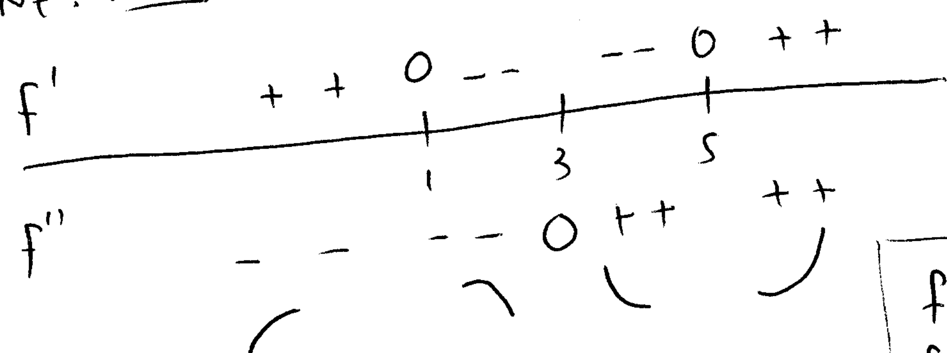
so f is increasing. If $a<b$ and $f(a)=f(b)=0$, then Rolle's Theorem says $f'(c)$ for some c b/w a & b, which is impossible, so f $\underline{\text{can't}}$ have two (or more) roots.

$\boxed{13.}$ $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x + 3$ $\quad f'(x) = x^2 - 6x + 5 = (x-1)(x-5) = 0$; $x = 1, 5$

($f'(x)$ DNE? $\underline{\text{never}}$)

$f''(x) = 2x - 6 = 0$; $x = 3$

($f''(x)$ DNE? $\underline{\text{never}}$.)



$\underline{\text{no}}$ vert. or horiz. asymptotes.

$f \nearrow: (-\infty, 1), (5, \infty)$

$f \searrow: (1, 5)$

f con $\nearrow$: $(3, \infty)$

f con $\searrow$: $(-\infty, 3)$

$f(1) = \frac{1}{3} - 3 + 5 + 3 = \frac{16}{3}$

$f(3) = 9 - 27 + 15 + 3 = 0$

$f(5) = \frac{125}{3} - 75 + 25 + 3$

$= 41 + \frac{2}{3} - 50 + 3$

$= \frac{2}{3} - 6 = -\frac{16}{3}$