Solutions

Name:

Math 106 Section 550 Exam 3

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it! You should solve each problem using the methods explored up to this point in class and the text, together with any necessary algebra, trigonometry, or geometry, as appropriate.

1. Use Newton's method to find approximations to a root of the function $f(x) = x^5 - x - 1$. Find x_2, starting with $x_0 = -1$.

$f(x) = x^5 - x - 1 \qquad f'(x) = 5x^4 - 1$

$x_0 = -1 \qquad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{((-1)^5 - (-1) - 1)}{5(-1)^4 - 1}$

$= -1 - \left(\frac{-1}{4}\right) = -1 + \frac{1}{4} = \boxed{\frac{-3}{4}}$

$x_2 = \boxed{\left(-\frac{3}{4}\right) - \frac{\left(\left(-\frac{3}{4}\right)^5 - \left(-\frac{3}{4}\right) - 1\right)}{5\left(-\frac{3}{4}\right)^4 - 1}}$

$= -\frac{3}{4} - \frac{\left(\frac{1}{4}\right)^5\left(-3^5 + 3\cdot 4^4 - 4^5\right)}{\left(\frac{1}{4}\right)^4\left(5\cdot 3^4 - 4^4\right)}$

$= -\frac{3}{4} - \frac{1}{4}\,\frac{-243 + 3\cdot 256 - 1024}{5\cdot 81 - 256} = -\frac{3}{4} - \frac{1}{4}\cdot\frac{768 - \cancel{244}\ 1267}{405 - 256}$

$= -\frac{3}{4} + \frac{1}{4}\,\frac{\cancel{524}\ 499}{149} = \frac{-3\cdot 149 + 499}{4\cdot \cancel{499}\ 149} = \frac{499 - 447}{4\cdot \cancel{499}\ 149}$

$= \frac{52}{4\cdot \cancel{499}\ 149} = \frac{13}{\cancel{499}\ 149}$

2. Find the following limits:

(a): $\lim\limits_{x\to1} \dfrac{\overset{\to 0}{x^3\ln x}}{\underset{\to 0}{x-1}} = \lim\limits_{x\to1} \dfrac{3x^2\ln x + x^3\left(\frac{1}{x}\right)}{1-0} = \lim\limits_{x\to1} 3x^2\ln x + x^2$

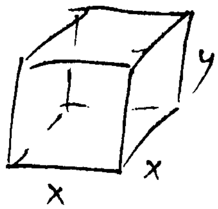
$= 3(1)^2(0)+(1)^2 = \boxed{1}$

(b): $\lim\limits_{x\to\infty} xe^{-x}\ln x = \lim\limits_{x\to\infty} \dfrac{\overset{\to 0}{x\ln x}}{e^x} = \lim\limits_{x\to\infty} \dfrac{\ln x + x\left(\frac{1}{x}\right)}{\underset{\to 0}{e^x}}$

$= \lim\limits_{x\to\infty} \dfrac{\overset{\to\infty}{\ln x+1}}{\underset{\to\infty}{e^x}} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}}{e^x} = \left(\dfrac{\text{small}}{\text{large}}\right) = \boxed{0}$

$= \lim\limits_{x\to\infty} \frac{1}{x} e^{-x} = 0\cdot 0 = 0.$

3. We wish to build a rectangular box with a square base and no top, out of materials that cost 5 cents per square inch for the bottom and 3 cents per square inch for the sides. What are the dimensions of the box with the largest volume that we can build for $3?



maximize volume $= x^2 y$

$$(5)(x^2) + 3(4xy) = 300$$

$$5x^2 + 12xy = 300$$

$$12xy = 300 - 5x^2$$

$$y = \frac{300 - 5x^2}{12x}$$

$x \geq 0$

$y \geq 0$

$\rightsquigarrow 300 - 5x^2 \geq 0$

$5x^2 \leq 300$

$x \leq \sqrt{60}$

$$\text{Volume} = V(x) = x^2 \frac{300 - 5x^2}{12x}$$

$$= \frac{1}{12} x (300 - 5x^2) = \frac{1}{12}(300x - 5x^3)$$

on $[0, \sqrt{60}]$

$$V'(x) = \frac{1}{12}(300 - 15x^2) = 0$$

$$15x^2 = 300$$

$$x^2 = 20$$

$$x = \sqrt{20}, \cancel{-\sqrt{20}}$$ not in domain

candidates! $x = 0, \sqrt{20}, \sqrt{60}$

$$V(0) = \frac{1}{12}(0)(300) = 0$$

$$V(\sqrt{20}) = \frac{1}{12}\sqrt{20}(300 - 100) = \frac{200\sqrt{20}}{12} \leftarrow \underline{\text{max}}$$

$$V(\sqrt{60}) = \frac{1}{12}\sqrt{60}(300 - 300) = 0$$

$$y = \frac{300 - 5(\sqrt{20})^2}{12\sqrt{20}} = \frac{200}{12\sqrt{20}}$$

$\boxed{x = \sqrt{20},\ y = \frac{200}{12\sqrt{20}}}$ make largest volume box.

4. Find the derivative of the function $H(x) = \int_{\ln x}^{3} \frac{\tan(e^t)}{t}\, dt$

$$H(x) = \int_{\ln x}^{3} \frac{\tan(e^t)}{t}\, dt = F(3) - F(\ln x)$$

where $F'(x) = \frac{\tan(e^x)}{x}$

$$H'(x) = 0 - (F'(\ln x))\left(\frac{1}{x}\right) = \boxed{-\frac{\tan(e^{\ln x})}{\ln x}\left(\frac{1}{x}\right)}$$

$$= \frac{\tan x}{x \ln x}$$

5. Find the following indefinite integrals:

(a): $\displaystyle\int \frac{x^5 - 5x^2 + 3}{x^2}\, dx$

$$= \int x^3 - 5 + \frac{3}{x^2}\, dx = \int x^3 - 5 + 3x^{-2}\, dx$$

$$= \boxed{\frac{x^4}{4} - 5x + 3\frac{x^{-1}}{-1} + C} = \frac{x^4}{4} - 5x - \frac{3}{x} + C$$

(b): $\displaystyle\int \sin^3 x \cos x\, dx$

$u = \sin x$
$du = \cos x\, dx$

$$= \int u^3\, du \Big|_{u=\sin x} = \frac{u^4}{4} + C \Big|_{u=\sin x} = \boxed{\frac{\sin^4 x}{4} + C}$$

6. Find the following definite integrals:

(a): $\int_0^{\pi} \sin x - \cos x dx$

$$= -\cos x - \sin x \Big|_0^{\pi} = (-\cos\pi - \sin\pi) - (-\cos 0 - \sin 0)$$

$$= (-(-1) - 0) - (-1 - 0) = 1 + 1 = \boxed{2}$$

(b): $\int_1^3 \frac{x}{(2x-1)^2} dx$

$u = 2x - 1 \qquad x = 1 \quad u = 2\cdot 1 - 1 = 1$

$du = 2dx \qquad x = 3 \quad u = 2\cdot 3 - 1 = 5$

$dx = \frac{1}{2} du \qquad u + 1 = 2x \quad x = \frac{u+1}{2}$

$$= \int_1^5 \frac{\frac{u+1}{2}}{u^2} \left(\frac{1}{2} du\right) = \frac{1}{4} \int_1^5 \frac{u+1}{u^2} du = \frac{1}{4} \int_1^5 u^{-1} + u^{-2} du$$

$$= \frac{1}{4}\left(\ln|u| + \frac{u^{-1}}{-1}\right)\Big|_1^5 = \frac{1}{4}\left(\ln|u| - u^{-1}\right)\Big|_1^5$$

$$= \frac{1}{4}\left(\left(\ln 5 - \frac{1}{5}\right) - \left(\ln 1 - \frac{1}{1}\right)\right) = \frac{1}{4}\left(\ln 5 - \frac{1}{5} - 0 + 1\right)$$

$$= \frac{1}{4}\left(\ln 5 + \frac{4}{5}\right) = \frac{1}{4}\ln 5 + \frac{1}{5}$$