

### Math 106 Section 550 Exam 3 Practice problems

The following problems will help you in preparing for the third exam, **in addition to** the problems assigned from the text for the sections covered by the exam (see the review sheet). There are more questions (perhaps twice as many) here than will appear on the exam.

You should show all of your work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it! You should solve each problem using the methods explored up to this point in class and the text, together with any necessary algebra, trigonometry, or geometry, as appropriate.

1. Use Newton's method to approximate  $\sqrt{18}$  (i.e., to approximate a solution to  $g(x) = x^2 - 18 = 0$ ); find  $x_2$ , starting from  $x_0 = 4$ .
2. Use Newton's method to approximate a root to  $g(x) = x^3 - x - 1 = 0$ ; find  $x_2$ , starting from  $x_0 = 1$ .
3. A line segment of length 6 has one endpoint on the (positive)  $x$ -axis and the other endpoint on the (positive)  $y$ -axis, forming a right triangle with the  $x$ - and  $y$ -axes. What is the largest area that this triangle can have?
4. You are given 500 feet of fencing to build a rectangular pen, which will be subdivided into 6 pieces, by adding two vertical and one horizontal fence lines. What is the largest area that can be enclosed?
5. Find the following limits:

$$\lim_{x \rightarrow 0} \frac{\tan x}{x}$$

$$\lim_{x \rightarrow 1} \frac{x^2 \ln x}{x^2 - 1}$$

$$\lim_{x \rightarrow 4} \frac{x^2 - 4}{x - 2}$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$$

$$\lim_{x \rightarrow \infty} \sqrt{x^2 - 3x + 9} - \sqrt{x^2 + 5x + 4}$$

$$\lim_{x \rightarrow 0^+} x \ln x$$

6. If  $G(x) = \int_x^{x^3} \frac{y^2 + 1}{y + 3} dy$ , find the derivative of  $G$ .

7. Find the derivative of  $F(x) = \int_1^{\ln x} \frac{e^t}{t} dt$ .

8. Find the average value of the function  $f(x) = x^3 + x + 3$  over the interval  $[2,4]$ .

9. Find the following definite and indefinite integrals:

$$\int \frac{1 - 3x}{1 + x^2} dx$$

$$\int (x^2 + 1)(x - 2) dx$$

$$\int_1^2 \frac{x^2}{x + 1} dx$$

$$\int_0^{\pi/6} \sqrt{1 + \sin x} dx$$

[Hint: note that  $\sqrt{1 + \sin x} = \frac{\cos x}{\sqrt{1 - \sin x}}$  .]

$$\int x^5 - 3x^4 + 14x^2 - 6x + 2 dx$$

$$\int (2x + 1)^{10} dx$$

$$\int \frac{(\ln x)^5}{x} dx$$

$$\int_0^1 x^2(1 - 2x^3)^8 dx$$

$$\int \frac{\cos x}{(1 + \sin x)^3} dx$$

$$\int_0^2 x^4 - 4x^2 - 1 dx$$