Math 106 Section 550 Exam 3 Practice Problems Solutions

1. $\sqrt{18}$ is a root of $f(x)=x^2-18$; $f'(x)=2x$ & for $x_0=4$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 4 - \frac{(-2)}{2(4)} = 4+\frac{1}{4} = \boxed{\frac{17}{4}}$$

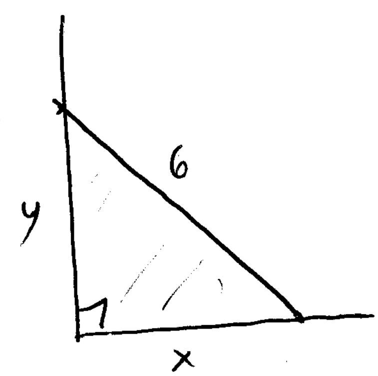
$$x_2 = \boxed{\frac{17}{4} - \frac{\left(\left(\frac{17}{4}\right)^2-18\right)}{\left(2\left(\frac{17}{4}\right)\right)}} = \text{some number.}$$

2. $g(x)=x^3-x-1$ $\quad g'(x)=3x^2-1$

For $x_0=1$, $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1 - \dfrac{(-1)}{(2)} = \boxed{\dfrac{3}{2}}$

$$x_2 = \frac{3}{2} - \frac{\left(\left(\frac{3}{2}\right)^3-\frac{3}{2}-1\right)}{3\left(\frac{3}{2}\right)^2-1} = \frac{3}{2} - \frac{\left(\frac{27}{8}-\frac{12}{8}-\frac{8}{8}\right)}{\left(\frac{27}{4}-\frac{4}{4}\right)} = \frac{3}{2} - \frac{\left(\frac{7}{8}\right)}{\frac{23}{4}}$$

$$= \frac{3}{2} - \frac{7}{46} = \frac{69-7}{46} = \frac{62}{46} = \boxed{\frac{31}{23}}$$

3. 

~~Minimize~~ Maximize Area $= \frac{1}{2}xy$

$x^2+y^2=6^2=36$; $y^2=36-x^2$, $y=\sqrt{36-x^2}$

Area $= A(x) = \frac{1}{2}x\sqrt{36-x^2}$ $\quad 0\le x\le 6$

$x\ge 0$ $\quad$ $y\ge 0$!

$$A'(x) = \frac{1}{2}\sqrt{36-x^2} + \frac{1}{2}x\left(\frac{1}{2}(36-x^2)^{-1/2}(-2x)\right)$$

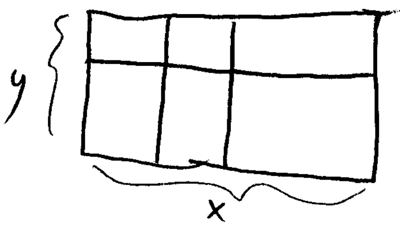
$$= \frac{1}{2\sqrt{36-x^2}}\left((36-x^2)-x^2\right) = \frac{36-2x^2}{2\sqrt{36-x^2}} = 0$$

$2x^2=36$
$x=\pm\sqrt{18}$

Candidates: $0, \sqrt{18}, \cancel{-\sqrt{18}}, 6$
not in domain

$A(0)=0$
$A(\sqrt{18}) = \frac{1}{2}\sqrt{18}\sqrt{18} = 9 \leftarrow$ max
$A(6)=0$

$\boxed{\text{Largest area} = 9}$

4. 

Area $= xy$

Fencing $= 3x + 4y = 500$; $y = \dfrac{500-3x}{4}$

Area $= A(x) = x\left(\dfrac{500-3x}{4}\right) = \dfrac{1}{4}\left(500x - 3x^2\right)$; $0 \le x \le \dfrac{500}{3}$ ($y \ge 0$!)

$A'(x) = \dfrac{1}{4}\left(500 - 6x\right) = 0 \qquad 6x = 500 \qquad x = \dfrac{500}{6} = \dfrac{250}{3}$

$\left(\left(y = \dfrac{500 - 3\left(\frac{250}{3}\right)}{4} = \dfrac{500-250}{4} = \dfrac{250}{4}\right)\right)$

Candidates: $x = 0, \dfrac{250}{3}, \dfrac{500}{3}$

$A(0) = 0$

$A\left(\frac{250}{3}\right) = \left(\frac{250}{3}\right)\left(\frac{250}{4}\right) \leftarrow$ max

$A\left(\frac{500}{3}\right) = \left(\frac{500}{3}\right)(0) = 0$

$\boxed{\text{Largest area} = \dfrac{(250)^2}{12}}$

5. $\lim\limits_{x\to 0} \dfrac{\tan x \ (\to 0)}{x \ (\to 0)} = \lim\limits_{x\to 0} \dfrac{(\tan x)'}{(x)'} = \lim\limits_{x\to 0} \dfrac{\sec^2 x}{1} = \dfrac{1}{1} = \boxed{1}$

$\lim\limits_{x\to 1} \dfrac{x^2 \ln x \ (\to 0)}{x^2 - 1 \ (\to 0)} = \lim\limits_{x\to 1} \dfrac{(x^2 \ln x)'}{(x^2-1)'} = \lim\limits_{x\to 1} \dfrac{2x \ln x + x^2\left(\frac{1}{x}\right)}{2x} = \dfrac{0+1}{2} = \boxed{\dfrac{1}{2}}$

$\lim\limits_{x\to 4} \dfrac{x^2-4}{x-2} = \dfrac{(4)^2-4}{4-2} = \dfrac{12}{2} = \boxed{6}$

$\lim\limits_{x\to 0} \dfrac{\cos x - 1 \ (\to 0)}{x^2 \ (\to 0)} = \lim\limits_{x\to 0} \dfrac{\sin x \ (\to 0)}{2x \ (\to 0)} = \lim\limits_{x\to 0} \dfrac{\cos x}{2} = \boxed{\dfrac{1}{2}}$

$\lim\limits_{x\to \infty} \sqrt{x^2-3x+9} - \sqrt{x^2+5x+4} = \lim\limits_{x\to\infty} \dfrac{(x^2-3x+9) - (x^2+5x+4)}{\sqrt{x^2-3x+9} + \sqrt{x^2+5x+4}}$

"$\infty - \infty$"

$= \lim\limits_{x\to\infty} \dfrac{-8x+5 \ (\to \infty)}{\sqrt{x^2-3x+9} + \sqrt{x^2+5x+4} \ (\to \infty)} = \lim\limits_{x\to\infty} \dfrac{-8}{\dfrac{2x-3}{2\sqrt{x^2-3x+9}} + \dfrac{2x+5}{2\sqrt{x^2+5x+4}}}$

$$= \lim_{x\to\infty} \frac{-8}{\frac{2-3/x}{2\sqrt{1-\frac{3}{x}+\frac{9}{x^2}}} + \frac{2+5/x}{2\sqrt{1+\frac{5}{x}+\frac{4}{x^2}}}} = \frac{-8}{\frac{2}{2}+\frac{2}{2}} = \frac{-8}{2} = \boxed{-4}.$$

$$\lim_{x\to 0^+} \overset{0}{x}\,\overset{-\infty}{\ln x} = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = \boxed{0}.$$

6. $G(x) = \int_x^{x^3} \frac{y^2+1}{y+3}\,dy = F(x^3) - F(x)$ for $F'(x) = \frac{x^2+1}{x+3}$

& $G'(x) = F'(x^3)[3x^2] - F'(x) = \boxed{\frac{(x^3)^2+1}{(x^3)+3}(3x^2) - \frac{x^2+1}{x+3}}.$

7. $F(x) = \int_1^{\ln x} \frac{e^t}{t}\,dt = G(\ln x) - G(1)$ for $G'(t) = \frac{e^t}{t}$.

& $F'(x) = G'(\ln x)\left(\frac{1}{x}\right) - 0 = \frac{e^{\ln x}}{\ln x}\left(\frac{1}{x}\right) = \frac{x}{\ln x}\left(\frac{1}{x}\right) = \boxed{\frac{1}{\ln x}}$

8. Avg value of $f(x) = x^3+x+3$ on $[2,4]$

$$= \frac{1}{4-2}\int_2^4 x^3+x+3\,dx = \frac{1}{2}\left[\frac{x^4}{4}+\frac{x^2}{2}+3x\right]\Big|_2^4$$

$$= \frac{1}{2}\Big((64+8+12)-(4+2+6)\Big) = \frac{1}{2}(84-12) = \frac{1}{2}(72) = \boxed{36}$$

9. $\int \frac{1-3x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx - 3\int \frac{x}{1+x^2}\,dx = \text{Arctan}x - \frac{3}{2}\int \frac{2x\,dx}{1+x^2}$

$u = 1+x^2$
$du = 2x\,dx$

$$= \text{Arctan}x - \frac{3}{2}\int \frac{du}{u}\Big|_{u=1+x^2} = \text{Arctan}x - \frac{3}{2}\ln|u| + C\Big|_{u=1+x^2}$$

$$\boxed{= \text{Arctan}x - \frac{3}{2}\ln|1+x^2| + C}$$

$$\int (x^2+1)(x-2)\,dx = \int x^3 + x - 2x^2 - 2\,dx = \boxed{\frac{x^4}{4} + \frac{x^2}{2} - \frac{2x^3}{3} - 2x + C.}$$

$$\int_1^2 \frac{x^2}{x+1}\,dx = \begin{pmatrix} u = x+1 \\ x = u-1 \\ x=1 \quad u=2 \\ x=2 \quad u=3 \end{pmatrix} = \int_2^3 \frac{(u-1)^2}{u}\,du = \int_2^3 \frac{u^2-2u+1}{u}\,du$$

$$= \int_2^3 u - 2 + \frac{1}{u}\,du = \frac{u^2}{2} - 2u + \ln|u| \Big|_2^3 = \boxed{\left(\frac{3^2}{2} - 6 + \ln(3)\right) - \left(\frac{2^2}{2} - 4 + \ln(2)\right)}$$

$$\int_0^{\pi/6} \sqrt{1+\sin x}\,dx = \int_0^{\pi/6} \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}\,dx = \int_0^{\pi/6} \frac{\cos x}{\sqrt{1-\sin x}}\,dx$$

$$\begin{pmatrix} u = 1-\sin x \\ du = -\cos x\,dx \\ x=0 \quad u=1 \\ x=\frac{\pi}{6} \quad u = 1-\frac{1}{2} = \frac{1}{2} \end{pmatrix} = \int_1^{1/2} \frac{-du}{\sqrt{u}} = -\int_1^{1/2} u^{-1/2}\,du = -\left(\frac{u^{1/2}}{1/2}\right)\Big|_1^{1/2}$$

$$= -\left(2\left(\tfrac{1}{2}\right)^{1/2} - 2(1)^{1/2}\right) = 2 - \frac{2}{\sqrt{2}} = \boxed{2-\sqrt{2}}.$$

$$\int x^5 - 3x^4 + 14x^2 - 6x + 2\,dx = \boxed{\frac{x^6}{6} - 3\left(\frac{x^5}{5}\right) + 14\left(\frac{x^3}{3}\right) - 6\left(\frac{x^2}{2}\right) + 2x + C.}$$

$$\int (2x+1)^{10}\,dx \begin{pmatrix} u = 2x+1 \\ du = 2\,dx \end{pmatrix} = \int u^{10}\left(\tfrac{1}{2}\,du\right)\Big|_{u=2x+1} = \frac{1}{2}\,\frac{u^{11}}{11} + C\Big|_{u=2x+1}$$

$$= \frac{1}{22}(2x+1)^{11} + C.$$

$$\int \frac{(\ln x)^5}{x}\,dx \begin{pmatrix} u = \ln x \\ du = \frac{1}{x}\,dx \end{pmatrix} = \int u^5\,du\Big|_{u=\ln x} = \frac{u^6}{6} + C\Big|_{u=\ln x} = \boxed{\frac{(\ln x)^6}{6} + C}$$

$$\int_0^1 x^2(1-2x^3)^8\,dx \quad \begin{pmatrix} u=1-2x^3 & x=0,\ u=1 \\ du=6x^2dx & x=1,\ u=-1 \\ x^2dx=\frac{1}{6}du & \end{pmatrix} = \frac{1}{6}\int_1^{-1} u^8\,du = \frac{1}{6}\frac{u^9}{9}\Big|_1^{-1}$$

$$= \frac{1}{54}\left((-1)^9-(1)^9\right) = \frac{-2}{54} = \boxed{\frac{-1}{27}}.$$

$$\int \frac{\cos x}{(1+\sin x)^3}\,dx = \begin{pmatrix} u=1+\sin x \\ du=\cos x\,dx \end{pmatrix} = \int \frac{du}{u^3}\Big|_{u=1+\sin x} = \int u^{-3}du\Big|_{u=1+\sin x} =$$

$$= \frac{u^{-2}}{-2}+C\Big|_{u=1+\sin x} = \frac{-1}{2u^2}+C\Big|_{u=1+\sin x} = \boxed{\frac{-1}{2(1+\sin x)^2}+C}$$

$$\int_0^2 x^4-4x^2-1\,dx = \frac{x^5}{5}-4\left(\frac{x^3}{3}\right)-x\Big|_0^2 = \left(\frac{2^5}{5}-4\left(\frac{2^3}{3}\right)-2\right)-0$$

$$= \frac{32}{5}-\frac{32}{3}-2 = \frac{96-160-30}{15} = \boxed{\frac{-94}{15}}$$