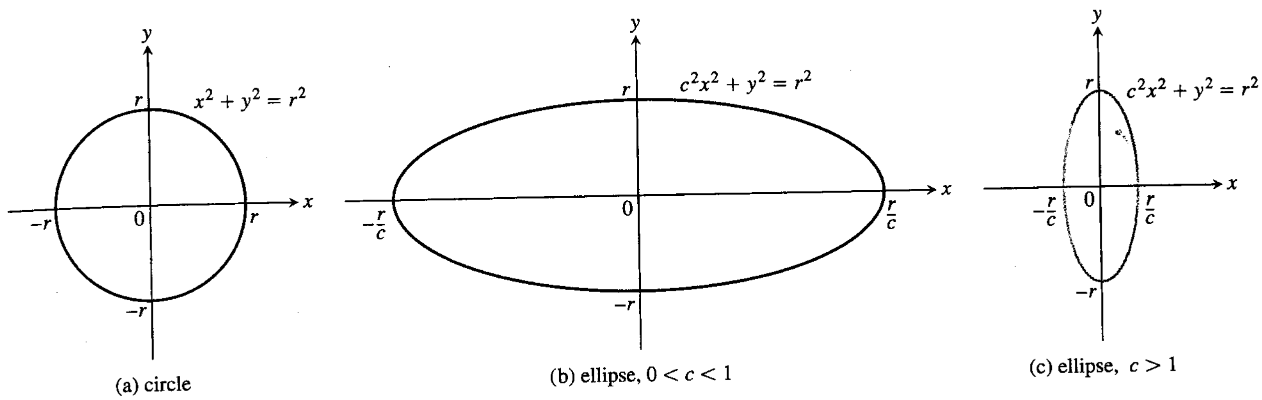


FIGURE 1.36 Horizontal stretchings or compressions of a circle produce graphs of ellipses.

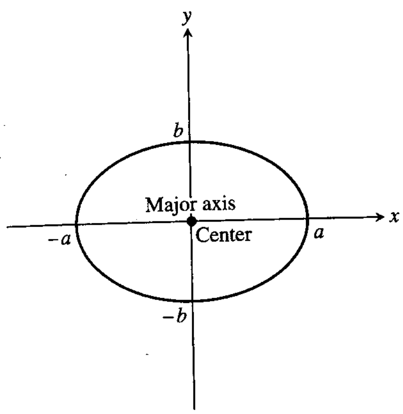


FIGURE 1.37 Graph of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, where the major axis is horizontal.

If $0 < c < 1$, the graph of Equation (1) horizontally stretches the circle; if $c > 1$ the circle is compressed horizontally. In either case, the graph of Equation (1) is an ellipse (Figure 1.36). Notice in Figure 1.36 that the y-intercepts of all three graphs are always $-r$ and r. In Figure 1.36b, the line segment joining the points $(\pm r/c, 0)$ is called the **major axis** of the ellipse; the **minor axis** is the line segment joining $(0, \pm r)$. The axes of the ellipse are reversed in Figure 1.36c: The major axis is the line segment joining the points $(0, \pm r)$, and the minor axis is the line segment joining the points $(\pm r/c, 0)$. In both cases, the major axis is the line segment having the longer length.

If we divide both sides of Equation (1) by r^2, we obtain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2}$$

where $a = r/c$ and $b = r$. If $a > b$, the major axis is horizontal; if $a < b$, the major axis is vertical. The **center** of the ellipse given by Equation (2) is the origin (Figure 1.37).

Substituting $x - h$ for x, and $y - k$ for y, in Equation (2) results in

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1. \tag{3}$$

Equation (3) is the **standard equation of an ellipse** with center at (h, k). The geometric definition and properties of ellipses are reviewed in Section 9.4.

EXERCISES 1.2

In Exercises 1 and 2, find the domains and ranges of f, g, $f + g$, and $f \cdot g$.

1. $f(x) = x, \quad g(x) = \sqrt{x-1}$

2. $f(x) = \sqrt{x+1}, \quad g(x) = \sqrt{x-1}$

In Exercises 3 and 4, find the domains and ranges of f, g, f/g, and g/f.

3. $f(x) = 2, \quad g(x) = x^2 + 1$

4. $f(x) = 1, \quad g(x) = 1 + \sqrt{x}$

5. If $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following.
   a. $f(g(0))$ b. $g(f(0))$
   c. $f(g(x))$ d. $g(f(x))$
   e. $f(f(-5))$ f. $g(g(2))$
   g. $f(f(x))$ h. $g(g(x))$
6. If $f(x) = x - 1$ and $g(x) = 1/(x + 1)$, find the following.
   a. $f(g(1/2))$ b. $g(f(1/2))$
   c. $f(g(x))$ d. $g(f(x))$
   e. $f(f(2))$ f. $g(g(2))$
   g. $f(f(x))$ h. $g(g(x))$
7. If $u(x) = 4x - 5$, $v(x) = x^2$, and $f(x) = 1/x$, find formulas for the following.
   a. $u(v(f(x)))$ b. $u(f(v(x)))$
   c. $v(u(f(x)))$ d. $v(f(u(x)))$
   e. $f(u(v(x)))$ f. $f(v(u(x)))$
8. If $f(x) = \sqrt{x}$, $g(x) = x/4$, and $h(x) = 4x - 8$, find formulas for the following.
   a. $h(g(f(x)))$ b. $h(f(g(x)))$
   c. $g(h(f(x)))$ d. $g(f(h(x)))$
   e. $f(g(h(x)))$ f. $f(h(g(x)))$

Let $f(x) = x - 3$, $g(x) = \sqrt{x}$, $h(x) = x^3$, and $j(x) = 2x$. Express each of the functions in Exercises 9 and 10 as a composite involving one or more of f, g, h, and j.

9. a. $y = \sqrt{x} - 3$ b. $y = 2\sqrt{x}$
   c. $y = x^{1/4}$ d. $y = 4x$
   e. $y = \sqrt{(x - 3)^3}$ f. $y = (2x - 6)^3$
10. a. $y = 2x - 3$ b. $y = x^{3/2}$
   c. $y = x^9$ d. $y = x - 6$
   e. $y = 2\sqrt{x - 3}$ f. $y = \sqrt{x^3 - 3}$
11. Copy and complete the following table.

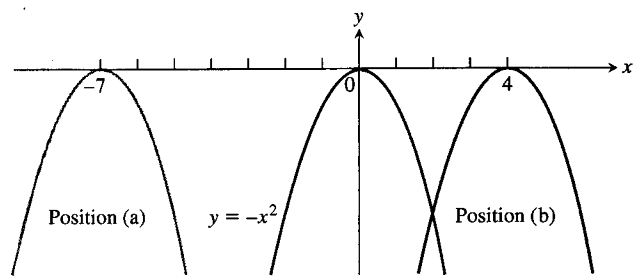
	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$x - 7$	$\sqrt{x}$	?
b.	$x + 2$	$3x$	?
c.	?	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
d.	$\frac{x}{x - 1}$	$\frac{x}{x - 1}$	?
e.	?	$1 + \frac{1}{x}$	x
f.	$\frac{1}{x}$	?	x

12. Copy and complete the following table.

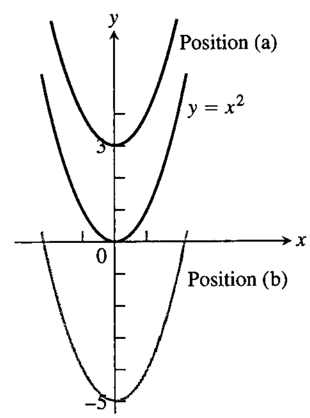
	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$\frac{1}{x - 1}$	$\lvert x\rvert$	?
b.	?	$\frac{x - 1}{x}$	$\frac{x}{x + 1}$
c.	?	$\sqrt{x}$	$\lvert x\rvert$
d.	$\sqrt{x}$	?	$\lvert x\rvert$

In Exercises 13 and 14, **(a)** write a formula for $f \circ g$ and $g \circ f$ and find the **(b)** domain and **(c)** range of each.

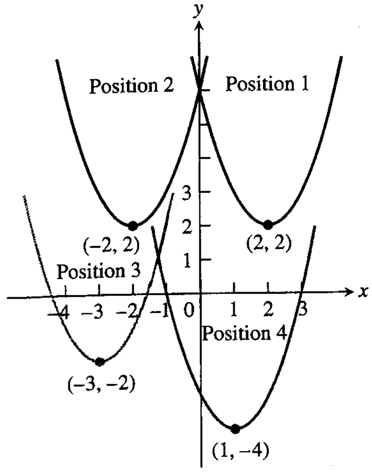
13. $f(x) = \sqrt{x + 1}$, $g(x) = \frac{1}{x}$
14. $f(x) = x^2$, $g(x) = 1 - \sqrt{x}$
15. The accompanying figure shows the graph of $y = -x^2$ shifted to two new positions. Write equations for the new graphs.



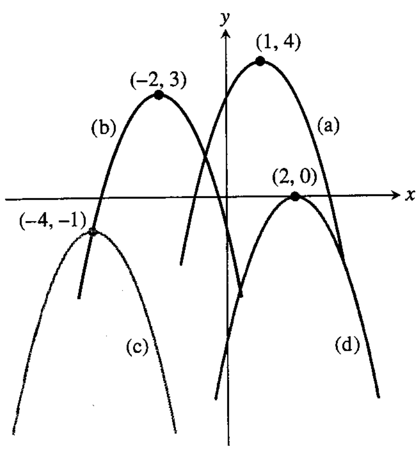
16. The accompanying figure shows the graph of $y = x^2$ shifted to two new positions. Write equations for the new graphs.



17. Match the equations listed in parts (a)–(d) to the graphs in the accompanying figure.
   a. $y = (x - 1)^2 - 4$ b. $y = (x - 2)^2 + 2$
   c. $y = (x + 2)^2 + 2$ d. $y = (x + 3)^2 - 2$



18. The accompanying figure shows the graph of $y = -x^2$ shifted to four new positions. Write an equation for each new graph.



Exercises 19–28 tell how many units and in what directions the graphs of the given equations are to be shifted. Give an equation for the shifted graph. Then sketch the original and shifted graphs together, labeling each graph with its equation.

19. $x^2 + y^2 = 49$ Down 3, left 2

20. $x^2 + y^2 = 25$ Up 3, left 4

21. $y = x^3$ Left 1, down 1

22. $y = x^{2/3}$ Right 1, down 1

23. $y = \sqrt{x}$ Left 0.81

24. $y = -\sqrt{x}$ Right 3

25. $y = 2x - 7$ Up 7

26. $y = \frac{1}{2}(x + 1) + 5$ Down 5, right 1

27. $y = 1/x$ Up 1, right 1

28. $y = 1/x^2$ Left 2, down 1

Graph the functions in Exercises 29–48.

29. $y = \sqrt{x + 4}$

30. $y = \sqrt{9 - x}$

31. $y = |x - 2|$

32. $y = |1 - x| - 1$

33. $y = 1 + \sqrt{x - 1}$

34. $y = 1 - \sqrt{x}$

35. $y = (x + 1)^{2/3}$

36. $y = (x - 8)^{2/3}$

37. $y = 1 - x^{2/3}$

38. $y + 4 = x^{2/3}$

39. $y = \sqrt[3]{x - 1} - 1$

40. $y = (x + 2)^{3/2} + 1$

41. $y = \dfrac{1}{x - 2}$

42. $y = \dfrac{1}{x} - 2$

43. $y = \dfrac{1}{x} + 2$

44. $y = \dfrac{1}{x + 2}$

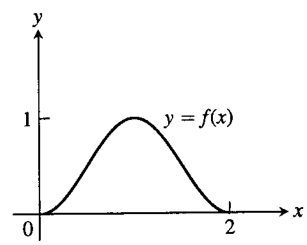
45. $y = \dfrac{1}{(x - 1)^2}$

46. $y = \dfrac{1}{x^2} - 1$

47. $y = \dfrac{1}{x^2} + 1$

48. $y = \dfrac{1}{(x + 1)^2}$

49. The accompanying figure shows the graph of a function $f(x)$ with domain $[0, 2]$ and range $[0, 1]$. Find the domains and ranges of the following functions, and sketch their graphs.



a. $f(x) + 2$

b. $f(x) - 1$

c. $2f(x)$

d. $-f(x)$

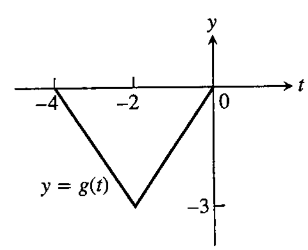
e. $f(x + 2)$

f. $f(x - 1)$

g. $f(-x)$

h. $-f(x + 1) + 1$

50. The accompanying figure shows the graph of a function $g(t)$ with domain $[-4, 0]$ and range $[-3, 0]$. Find the domains and ranges of the following functions, and sketch their graphs.



a. $g(-t)$

b. $-g(t)$

c. $g(t) + 3$

d. $1 - g(t)$

e. $g(-t + 2)$

f. $g(t - 2)$

g. $g(1 - t)$

h. $-g(t - 4)$

Exercises 51–60 tell by what factor and direction the graphs of the given functions are to be stretched or compressed. Give an equation for the stretched or compressed graph.

51. $y = x^2 - 1$, stretched vertically by a factor of 3

52. $y = x^2 - 1$, compressed horizontally by a factor of 2

53. $y = 1 + \frac{1}{x^2}$, compressed vertically by a factor of 2

54. $y = 1 + \frac{1}{x^2}$, stretched horizontally by a factor of 3

55. $y = \sqrt{x + 1}$, compressed horizontally by a factor of 4

56. $y = \sqrt{x + 1}$, stretched vertically by a factor of 3

57. $y = \sqrt{4 - x^2}$, stretched horizontally by a factor of 2

58. $y = \sqrt{4 - x^2}$, compressed vertically by a factor of 3

59. $y = 1 - x^3$, compressed horizontally by a factor of 3

60. $y = 1 - x^3$, stretched horizontally by a factor of 2

In Exercises 61–68, graph each function, not by plotting points, but by starting with the graph of one of the standard functions presented in Figures 1.13–1.15, and applying an appropriate transformation.

61. $y = -\sqrt{2x + 1}$ **62.** $y = \sqrt{1 - \frac{x}{2}}$

63. $y = (x - 1)^3 + 2$ **64.** $y = (1 - x)^3 + 2$

65. $y = \frac{1}{2x} - 1$ **66.** $y = \frac{2}{x^2} + 1$

67. $y = -\sqrt[3]{x}$ **68.** $y = (-2x)^{2/3}$

69. Graph the function $y = |x^2 - 1|$.

70. Graph the function $y = \sqrt{|x|}$.

Exercises 71–76 give equations of ellipses. Put each equation in standard form and sketch the ellipse.

71. $9x^2 + 25y^2 = 225$ **72.** $16x^2 + 7y^2 = 112$

73. $3x^2 + (y - 2)^2 = 3$ **74.** $(x + 1)^2 + 2y^2 = 4$

75. $3(x - 1)^2 + 2(y + 2)^2 = 6$

76. $6\left(x + \frac{3}{2}\right)^2 + 9\left(y - \frac{1}{2}\right)^2 = 54$

77. Write an equation for the ellipse $(x^2/16) + (y^2/9) = 1$ shifted 4 units to the left and 3 units up. Sketch the ellipse and identify its center and major axis.

78. Write an equation for the ellipse $(x^2/4) + (y^2/25) = 1$ shifted 3 units to the right and 2 units down. Sketch the ellipse and identify its center and major axis.

79. Assume that f is an even function, g is an odd function, and both f and g are defined on the entire real line $\mathbb{R}$. Which of the following (where defined) are even? odd?

a. fg **b.** f/g **c.** g/f

d. $f^2 = ff$ **e.** $g^2 = gg$ **f.** $f \circ g$

g. $g \circ f$ **h.** $f \circ f$ **i.** $g \circ g$

80. Can a function be both even and odd? Give reasons for your answer.

T 81. (*Continuation of Example* 1.) Graph the functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1 - x}$ together with their (a) sum, (b) product, (c) two differences, (d) two quotients.

T 82. Let $f(x) = x - 7$ and $g(x) = x^2$. Graph f and g together with $f \circ g$ and $g \circ f$.

1.3 Trigonometric Functions

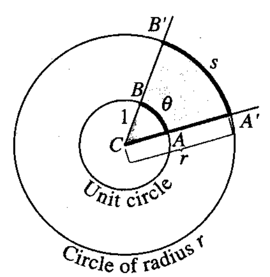


FIGURE 1.38 The radian measure of the central angle $A'CB'$ is the number $\theta = s/r$. For a unit circle of radius $r = 1$, θ is the length of arc AB that central angle ACB cuts from the unit circle.

This section reviews radian measure and the basic trigonometric functions.

Angles

Angles are measured in degrees or radians. The number of **radians** in the central angle $A'CB'$ within a circle of radius r is defined as the number of "radius units" contained in the arc s subtended by that central angle. If we denote this central angle by θ when measured in radians, this means that $\theta = s/r$ (Figure 1.38), or

$$s = r\theta \qquad (\theta \text{ in radians}). \tag{1}$$

If the circle is a unit circle having radius $r = 1$, then from Figure 1.38 and Equation (1), we see that the central angle θ measured in radians is just the length of the arc that the angle cuts from the unit circle. Since one complete revolution of the unit circle is 360° or 2π radians, we have

$$\pi \text{ radians} = 180^\circ \tag{2}$$