EXERCISES 2.5

Find the limits in Exercises 1–12.

1. $\lim_{x\to 0^+} \frac{1}{3x}$

2. $\lim_{x\to 0^-} \frac{5}{2x}$

3. $\lim_{x\to 2^-} \frac{3}{x-2}$

4. $\lim_{x\to 3^+} \frac{1}{x-3}$

5. $\lim_{x\to -8^+} \frac{2x}{x+8}$

6. $\lim_{x\to -5^-} \frac{3x}{2x+10}$

7. $\lim_{x\to 7} \frac{4}{(x-7)^2}$

8. $\lim_{x\to 0} \frac{-1}{x^2(x+1)}$

9. a. $\lim_{x\to 0^+} \frac{2}{3x^{1/3}}$ b. $\lim_{x\to 0^-} \frac{2}{3x^{1/3}}$

10. a. $\lim_{x\to 0^+} \frac{2}{x^{1/5}}$ b. $\lim_{x\to 0^-} \frac{2}{x^{1/5}}$

11. $\lim_{x\to 0} \frac{4}{x^{2/5}}$

12. $\lim_{x\to 0} \frac{1}{x^{2/3}}$

Find the limits in Exercises 13–16.

13. $\lim_{x\to(\pi/2)^-} \tan x$

14. $\lim_{x\to(-\pi/2)^+} \sec x$

15. $\lim_{\theta\to 0^-} (1 + \csc\theta)$

16. $\lim_{\theta\to 0} (2 - \cot\theta)$

Find the limits in Exercises 17–22.

17. $\lim \frac{1}{x^2-4}$ as
 a. $x\to 2^+$ b. $x\to 2^-$
 c. $x\to -2^+$ d. $x\to -2^-$

18. $\lim \frac{x}{x^2-1}$ as
 a. $x\to 1^+$ b. $x\to 1^-$
 c. $x\to -1^+$ d. $x\to -1^-$

19. $\lim\left(\frac{x^2}{2} - \frac{1}{x}\right)$ as
 a. $x\to 0^+$ b. $x\to 0^-$
 c. $x\to \sqrt[3]{2}$ d. $x\to -1$

20. $\lim \frac{x^2-1}{2x+4}$ as
 a. $x\to -2^+$ b. $x\to -2^-$
 c. $x\to 1^+$ d. $x\to 0^-$

21. $\lim \frac{x^2-3x+2}{x^3-2x^2}$ as
 a. $x\to 0^+$ b. $x\to 2^+$
 c. $x\to 2^-$ d. $x\to 2$
 e. What, if anything, can be said about the limit as $x\to 0$?

22. $\lim \frac{x^2-3x+2}{x^3-4x}$ as
 a. $x\to 2^+$ b. $x\to -2^+$
 c. $x\to 0^-$ d. $x\to 1^+$
 e. What, if anything, can be said about the limit as $x\to 0$?

Find the limits in Exercises 23–26.

23. $\lim\left(2 - \frac{3}{t^{1/3}}\right)$ as
 a. $t\to 0^+$ b. $t\to 0^-$

24. $\lim\left(\frac{1}{t^{3/5}} + 7\right)$ as
 a. $t\to 0^+$ b. $t\to 0^-$

25. $\lim\left(\frac{1}{x^{2/3}} + \frac{2}{(x-1)^{2/3}}\right)$ as
 a. $x\to 0^+$ b. $x\to 0^-$
 c. $x\to 1^+$ d. $x\to 1^-$

26. $\lim\left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right)$ as
 a. $x\to 0^+$ b. $x\to 0^-$
 c. $x\to 1^+$ d. $x\to 1^-$

Graph the rational functions in Exercises 27–38. Include the graphs and equations of the asymptotes.

27. $y = \frac{1}{x-1}$

28. $y = \frac{1}{x+1}$

29. $y = \frac{1}{2x+4}$

30. $y = \frac{-3}{x-3}$

31. $y = \frac{x+3}{x+2}$

32. $y = \frac{2x}{x+1}$

33. $y = \frac{x^2}{x-1}$

34. $y = \frac{x^2+1}{x-1}$

35. $y = \frac{x^2-4}{x-1}$

36. $y = \frac{x^2-1}{2x+4}$

37. $y = \frac{x^2-1}{x}$

38. $y = \frac{x^3+1}{x^2}$

In Exercises 39–42, sketch the graph of a function $y = f(x)$ that satisfies the given conditions. No formulas are required—just label the coordinate axes and sketch an appropriate graph. (The answers are not unique, so your graphs may not be exactly like those in the answer section.)

39. $f(0) = 0, f(1) = 2, f(-1) = -2, \lim_{x\to-\infty} f(x) = -1$, and $\lim_{x\to\infty} f(x) = 1$

40. $f(0) = 0, \lim_{x\to\pm\infty} f(x) = 0, \lim_{x\to 0^+} f(x) = 2$, and $\lim_{x\to 0^-} f(x) = -2$

41. $f(0) = 0$, $\lim_{x\to\pm\infty} f(x) = 0$, $\lim_{x\to 1^-} f(x) = \lim_{x\to -1^+} f(x) = \infty$, $\lim_{x\to 1^+} f(x) = -\infty$, and $\lim_{x\to -1^-} f(x) = -\infty$

42. $f(2) = 1, f(-1) = 0$, $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to 0^+} f(x) = \infty$, $\lim_{x\to 0^-} f(x) = -\infty$, and $\lim_{x\to -\infty} f(x) = 1$

In Exercises 43–46, find a function that satisfies the given conditions and sketch its graph. (The answers here are not unique. Any function that satisfies the conditions is acceptable. Feel free to use formulas defined in pieces if that will help.)

43. $\lim_{x\to\pm\infty} f(x) = 0$, $\lim_{x\to 2^-} f(x) = \infty$, and $\lim_{x\to 2^+} f(x) = \infty$

44. $\lim_{x\to\pm\infty} g(x) = 0$, $\lim_{x\to 3^-} g(x) = -\infty$, and $\lim_{x\to 3^+} g(x) = \infty$

45. $\lim_{x\to -\infty} h(x) = -1$, $\lim_{x\to\infty} h(x) = 1$, $\lim_{x\to 0^-} h(x) = -1$, and $\lim_{x\to 0^+} h(x) = 1$

46. $\lim_{x\to\pm\infty} k(x) = 1$, $\lim_{x\to 1^-} k(x) = \infty$, and $\lim_{x\to 1^+} k(x) = -\infty$

Use formal definitions to prove the limit statements in Exercises 47–50.

47. $\lim_{x\to 0} \dfrac{-1}{x^2} = -\infty$

48. $\lim_{x\to 0} \dfrac{1}{|x|} = \infty$

49. $\lim_{x\to 3} \dfrac{-2}{(x-3)^2} = -\infty$

50. $\lim_{x\to -5} \dfrac{1}{(x+5)^2} = \infty$

51. Here is the definition of **infinite right-hand limit**.

> We say that $f(x)$ approaches infinity as x approaches x_0 from the right, and write
>
> $$\lim_{x\to x_0^+} f(x) = \infty,$$
>
> if, for every positive real number B, there exists a corresponding number $\delta > 0$ such that for all x
>
> $$x_0 < x < x_0 + \delta \quad \Rightarrow \quad f(x) > B.$$

Modify the definition to cover the following cases.

a. $\lim_{x\to x_0^-} f(x) = \infty$

b. $\lim_{x\to x_0^+} f(x) = -\infty$

c. $\lim_{x\to x_0^-} f(x) = -\infty$

Use the formal definitions from Exercise 51 to prove the limit statements in Exercises 52–56.

52. $\lim_{x\to 0^+} \dfrac{1}{x} = \infty$

53. $\lim_{x\to 0^-} \dfrac{1}{x} = -\infty$

54. $\lim_{x\to 2^-} \dfrac{1}{x-2} = -\infty$

55. $\lim_{x\to 2^+} \dfrac{1}{x-2} = \infty$

56. $\lim_{x\to 1^-} \dfrac{1}{1-x^2} = \infty$

Each of the functions in Exercises 57–60 is given as the sum or difference of two terms. First graph the terms (with the same set of axes). Then, using these graphs as guides, sketch in the graph of the function.

57. $y = \sec x + \dfrac{1}{x}, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

58. $y = \sec x - \dfrac{1}{x^2}, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

59. $y = \tan x + \dfrac{1}{x^2}, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

60. $y = \dfrac{1}{x} - \tan x, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

T Graph the curves in Exercises 61–64. Explain the relation between the curve's formula and what you see.

61. $y = \dfrac{x}{\sqrt{4-x^2}}$

62. $y = \dfrac{-1}{\sqrt{4-x^2}}$

63. $y = x^{2/3} + \dfrac{1}{x^{1/3}}$

64. $y = \sin\left(\dfrac{\pi}{x^2+1}\right)$

T In Exercises 65–68, use the graph of $y = f(1/x)$ to find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

65. $f(x) = xe^x$

66. $f(x) = x^2e^{-x}$

67. $f(x) = \dfrac{\ln|x|}{x}$

68. $f(x) = \dfrac{e^{1/x}}{\ln|x|}$

T Graph the functions in Exercises 69 and 70. Then answer the following questions.

a. How does the graph behave as $x \to 0^+$?

b. How does the graph behave as $x \to \pm\infty$?

c. How does the graph behave at $x = 1$ and $x = -1$?

Give reasons for your answers.

69. $y = \dfrac{3}{2}\left(x - \dfrac{1}{x}\right)^{2/3}$

70. $y = \dfrac{3}{2}\left(\dfrac{x}{x-1}\right)^{2/3}$

2.6 Continuity

When we plot function values generated in a laboratory or collected in the field, we often connect the plotted points with an unbroken curve to show what the function's values are likely to have been at the times we did not measure (Figure 2.53). In doing so, we are