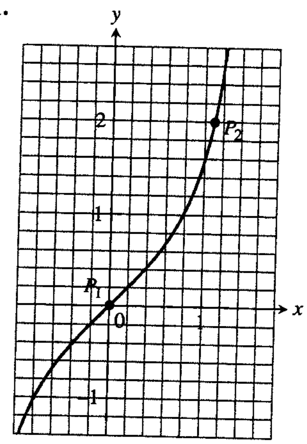
EXERCISES 2.7

In Exercises 1–4, use the grid and a straight edge to make a rough estimate of the slope of the curve (in y-units per x-unit) at the points P_1 and P_2. Graphs can shift during a press run, so your estimates may be somewhat different from those in the back of the book.

1.



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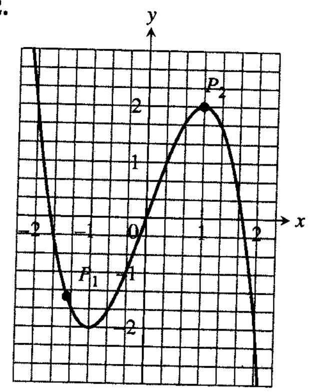

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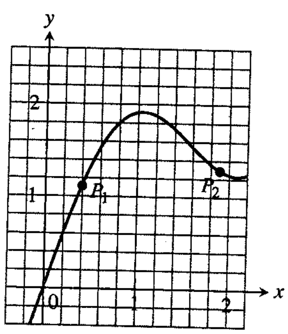

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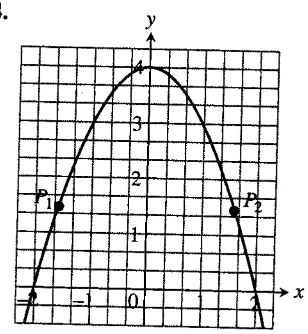


In Exercises 5–10, find an equation for the tangent to the curve at the given point. Then sketch the curve and tangent together.

5. $y = 3 - x^2$, $(-1, 2)$ **6.** $y = (x - 1)^2 + 1$, $(1, 1)$

7. $y = 2\sqrt{x}$, $(1, 2)$ **8.** $y = \dfrac{1}{x^2}$, $(-1, 1)$

9. $y = x^3 + 1$, $(-2, -7)$ **10.** $y = \dfrac{1}{x^3}$, $\left(-2, -\dfrac{1}{8}\right)$

In Exercises 11–18, find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

11. $f(x) = x^2 + 1$, $(2, 5)$ **12.** $f(x) = x - 2x^2$, $(1, -1)$

13. $g(x) = \dfrac{x}{x - 2}$, $(3, 3)$ **14.** $g(x) = \dfrac{8}{x^2}$, $(2, 2)$

15. $h(t) = t^3 - t$, $(2, 6)$ **16.** $h(t) = t^3 + 3t$, $(1, 4)$

17. $f(x) = \sqrt{x}$, $(4, 2)$ **18.** $f(x) = \sqrt{x + 1}$, $(8, 3)$

In Exercises 19–22, find the slope of the curve at the point indicated

19. $y = 5x^2$, $x = -1$ **20.** $y = 1 - x^2$, $x = 2$

21. $y = \dfrac{1}{x - 1}$, $x = 3$ **22.** $y = \dfrac{x - 1}{x + 1}$, $x = 0$

At what points do the graphs of the functions in Exercises 23 and 2[illegible] have horizontal tangents?

23. $f(x) = x^2 + 4x - 1$ **24.** $g(x) = x^3 - 3x$

25. Find equations of all lines having slope -1 that are tangent to th[e] curve $y = 1/(x - 1)$.

26. Find an equation of the straight line having slope $1/4$ that is tan[-] gent to the curve $y = \sqrt{x}$.

27. Object dropped from a tower An object is dropped from th[e] top of a 100-m-high tower. Its height above ground after t sec [is] $100 - 4.9t^2$ m. How fast is it falling 2 sec after it is dropped?

28. Speed of a rocket At t sec after liftoff, the height of a rocket [is] $3t^2$ ft. How fast is the rocket climbing 10 sec after liftoff?

29. Circle's changing area What is the rate of change of the area o[f] a circle ($A = \pi r^2$) with respect to the radius when the radius [is] $r = 3$?

30. Ball's changing volume What is the rate of change of the vol[-] ume of a ball ($V = (4/3)\pi r^3$) with respect to the radius when th[e] radius is $r = 2$?

31. Show that the line $y = mx + b$ is its own tangent at any poi[nt] $(x_0, mx_0 + b)$.

32. Find the slope of the tangent to the curve $y = 1/\sqrt{x}$ at the poi[nt] where $x = 4$.

33. Does the graph of

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

have a tangent at the origin? Give reasons for your answer.

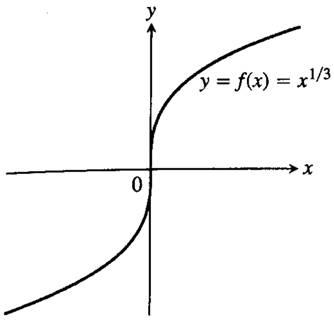
34. Does the graph of

$$g(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

have a tangent at the origin? Give reasons for your answer.

Vertical Tangents We say that the curve $y = f(x)$ has a **vertical tan[-] gent** at the point where $x = x_0$ if $\lim_{h\to 0} (f(x_0 + h) - f(x_0))/h = \infty$ or $-\infty$. Vertical tangent at $x = 0$ (see accompanying figure):

$$\lim_{h\to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h\to 0} \frac{h^{1/3} - 0}{h}$$
$$= \lim_{h\to 0} \frac{1}{h^{2/3}} = \infty$$

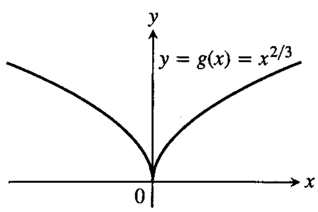


VERTICAL TANGENT AT ORIGIN

No vertical tangent at $x = 0$ (see next figure):

$$\lim_{h \to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h \to 0} \frac{h^{2/3} - 0}{h}$$
$$= \lim_{h \to 0} \frac{1}{h^{1/3}}$$

does not exist, because the limit is ∞ from the right and $-\infty$ from the left.



NO VERTICAL TANGENT AT ORIGIN

35. Does the graph of

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

have a vertical tangent at the origin? Give reasons for your answer.

36. Does the graph of

$$U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

have a vertical tangent at the point (0, 1)? Give reasons for your answer.

T **a.** Graph the curves in Exercises 37–46. Where do the graphs appear to have vertical tangents?

b. Confirm your findings in part (a) with limit calculations. But before you do, read the introduction to Exercises 35 and 36.

37. $y = x^{2/5}$

38. $y = x^{4/5}$

39. $y = x^{1/5}$

40. $y = x^{3/5}$

41. $y = 4x^{2/5} - 2x$

42. $y = x^{5/3} - 5x^{2/3}$

43. $y = x^{2/3} - (x-1)^{1/3}$

44. $y = x^{1/3} + (x-1)^{1/3}$

45. $y = \begin{cases} -\sqrt{|x|}, & x \leq 0 \\ \sqrt{x}, & x > 0 \end{cases}$

46. $y = \sqrt{|4 - x|}$

COMPUTER EXPLORATIONS

Use a CAS to perform the following steps for the functions in Exercises 47–50.

a. Plot $y = f(x)$ over the interval $(x_0 - 1/2) \leq x \leq (x_0 + 3)$.

b. Holding x_0 fixed, the difference quotient

$$q(h) = \frac{f(x_0 + h) - f(x_0)}{h}$$

at x_0 becomes a function of the step size h. Enter this function into your CAS workspace.

c. Find the limit of q as $h \to 0$.

d. Define the secant lines $y = f(x_0) + q \cdot (x - x_0)$ for $h = 3, 2,$ and 1. Graph them together with f and the tangent line over the interval in part (a).

47. $f(x) = x^3 + 2x, \quad x_0 = 0$

48. $f(x) = x + \frac{5}{x}, \quad x_0 = 1$

49. $f(x) = x + \sin(2x), \quad x_0 = \pi/2$

50. $f(x) = \cos x + 4\sin(2x), \quad x_0 = \pi$

Chapter 2 Questions to Guide Your Review

1. What is the average rate of change of the function $g(t)$ over the interval from $t = a$ to $t = b$? How is it related to a secant line?
2. What limit must be calculated to find the rate of change or slope of a function $g(t)$ at $t = t_0$?
3. What is an informal or intuitive definition of the limit

$$\lim_{x \to x_0} f(x) = L?$$

Why is the definition "informal"? Give examples.

4. Does the existence and value of the limit of a function $f(x)$ as x approaches x_0 ever depend on what happens at $x = x_0$? Explain and give examples.
5. What function behaviors might occur for which the limit may fail to exist? Give examples.
6. What theorems are available for calculating limits? Give examples of how the theorems are used.