

Math 106, Fall 2007
Group Project — “The Big Race”
Assigned: October 11, 2007
Due Date: November 15, 2007

Guidelines: Kepp in mind that your grade is based on both the accuracy of your answers and the quality of your written presentation. The paper you turn in should have a mix of equations, formulas, and prose. Graphs may be copied from your calculator or computer, but should be clearly labeled. Use complete sentences, good grammar, and correct punctuation and spelling. Justify your answers as much as possible. A good guideline is that your report should be written so that it could be understood by another Math 106 student who has not necessarily worked on this project.

Marna is a bicycle racing star and today she is in a big race. Moving at a constant velocity of k meters per second, she passes a refreshment station. At this instant, which we will take to be $t = 0$, her support car starts from the refreshment station to accelerate after her, beginning from a dead stop, i.e., velocity zero.

Since Marna is traveling at a constant velocity k m/sec, the distance traveled by Marna in t seconds is given by the function $d(t) = kt$. Suppose that the distance covered by the support car is given by the function $s(t) = 6t^2 - t^3$, where the distance is measured in meters and time in seconds. The function $s(t)$ is carefully calculated by her crew so that at the instance the car catches up to the racer, they will be traveling at the same velocity and a crew member will hand Marna a cold drink. The support car will then immediately fall behind.

On the basis of the above information, answer the following questions:

1. How fast is Marna traveling? How long does it take for the support car to catch her?
2. Now suppose Marna is riding at a constant velocity v (which for the purposes of this part of the project may be different from k).
 - (a) Find an expression that gives the time (or times) when the car and the bike meet, in terms of v .
 - (b) How many times would the car and the bike meet if v is greater than k ?
 - (c) How many times would the car and the bike meet if v is less than k ?
3. Consider a pair of axes with time measured horizontally and distance vertically. Draw graphs that depict the distance traveled by Marna and by the support car on the same axes. You should have four graphs; one for the function $s(t)$ and three for the function $d(t)$ corresponding to the cases $v = k$, $v > k$, and $v < k$.
4. If Marna had been traveling with a velocity v which was either greater than or less than (but not equal to) k , her crew feels that passing the drink would have been less easy. Why? Explain your answer.
5. Now answer the following theoretical problem: Prove that if a cubic polynomial $P(x)$ has a double root at $x = a$, then the derivative of $P(x)$ evaluated at $x = a$ must be zero. How does this relate to your answers to some of the parts above?