Math 107H

Practice problems for exam 3

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

12. Does the integral $\displaystyle\int_1^\infty \frac{1}{e^x - x}\,\mathrm{d}x$ converge or diverge?

In problems 1, 2, and 3, find the indicated limits.

1. $\displaystyle\lim_{n\to\infty} \frac{1 - 4n^3 + 4n}{3n^2 - n + 3} =$
2. $\displaystyle\lim_{n\to\infty} \frac{n^{n+\frac{1}{n}}}{(n+2)^n} =$
3. $\displaystyle\lim_{n\to\infty} \frac{n^2 + 4n\cos n - 1}{2n^2 + 15} =$
4. Find the indicated sum: $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 1 + 3^n}{5^n} =$

Using any (legal) method (other than psychic powers), determine the convergence or divergence of the following infinite series (be sure to show sufficient work so that the method used in determining conv/div can be understood):

5. $\displaystyle\sum_{n=0}^{\infty} \frac{n-3}{n+1}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + n}{n!}$
7. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{2/3}}$
8. $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{(\ln n)^n}$
9. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^9 + 3n - 4}}$
10. $\displaystyle\sum_{n=0}^{\infty} ne^{-n^2}$
11. $\displaystyle\sum_{n=3}^{\infty} \frac{n}{n^2 - 6}$

12. $\displaystyle\sum_{n=0}^{\infty} \frac{2n^3 - 1}{5^n}$

5. Find the radius of convergence of the following power series:

   (a): $\displaystyle\sum_{n=0}^{\infty} \frac{n5^n}{n!} x^n$

   (b): $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n - 1} (x-2)^n$

6. Find the Taylor polynomial, of degree 3, centered at a=0, for the function

$$f(x) = (x+3)^{5/2}$$

7. Find the power series (centered at 0) for the function

$$f(x) = \frac{1}{(1+x^2)^2}$$

   (Hint: Start with a series for $\dfrac{1}{1-x}$, and build from there... (derivative? integral?))