Math 107H, Section 3

Practice Problems for Exam 3

1. Show that the alternating series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ converges, and determine a value of N so that the partial sum $\displaystyle\sum_{n=2}^{N} \frac{(-1)^n}{\ln(n)}$ is within .001 of the infinite sum.

2. Compute the radius of convergence of the following power series:

$$f(x) = \sum_{n=0}^{\infty} \frac{2^n - 1}{(n+4)^2}(x-3)^n = \sum a_n(x-3)^n$$

3. Using the Taylor series $\displaystyle\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, find a power series representation for the function

$$f(x) = \frac{x^2}{1+x^4}$$

centered at $x = 0$ (by an appropriate substitution and multiplication). Use this to find a series which converges to the integral

$$\int_0^{1/3} \frac{x^2}{1+x^4}\, dx \ .$$

4. Find the Taylor polynomial of degree 3, centered at $x = 8$, for the function

$$f(x) = x^{2/3}$$

and estimate the error in using your polynomial to approximate $f(7) = 7^{2/3}$.

5. Express the polar equation $r = \sin(3\theta)$ as an equation in Cartesian coordinates. [Hint: $\sin(3\theta) = \sin(\theta = 2\theta)$...]

6. Find the (rectangular) equation of the line tangent to the graph of the polar curve

$$r = 3\sin\theta - \cos(3\theta)$$

at the point where $\theta = \dfrac{\pi}{4}$.

7. Find the length of the polar curve $r = \theta^2$ from $\theta = 0$ to $\theta = 2\pi$.

8. Find the area inside of the graph of the polar curve

$$r = \sin(\theta) - \cos(\theta)$$

from $\theta = \dfrac{\pi}{4}$ to $\theta = \dfrac{5\pi}{4}$.

[Extra credit: What does this curve look like? (Hint: multiply both sides by r.)]