Name:

Math 107H, Section 3

Exam 3

This take-home exam is due **at the beginning of class** at 9:30am on Wednesday, December 1, 2010. Early submission of your exam paper is welcome.

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

You may not discuss the methods used or details of the solutions to the problems on this exam with any person or entity other than your instructor, until you have turned in your exam or the beginning of our Wednesday class, whichever is later.

You may consult your notes, the typed review materials provided on our class website, and our textbook. You may consult no other sources as part of your work on the problems on this exam.

For questions requiring a numerical answer, it is acceptable (and even preferred) to leave your answer as an expression involving functions such as $\sqrt{x}$, e^x, $\ln x$, $\arctan x$, and the like. [Values of standard trigonometric functions at the standard angles should be computed as exact values.]

Best practices: We will attempt to arrange time at the end of class Monday for you to be able to read over the problems on the exam and be certain that you understand what is being asked of you on each question. If something seems unclear to you, **ask your instructor** (in person or by email), do not ask someone else! Your instructor is the final authority on how a question should be interpreted. Be sure to budget adequate time, in advance of the time the exam is due, to work the problems to your satisfaction. You are not required to work all of the problems in a single sitting, although that is probably the optimal strategy. In any event, make sure to find time **early in the exam period** to begin work on the exam. We expect that this exam should take an amount of time comparable to our first two exams to complete, which for most of you should translate to between 50 and 100 minutes. Your individual time may, however, vary.

1. Show that the alternating series $\sum_{n=2}^{\infty}(-1)^n \ln\left(\frac{n+1}{n}\right)$ converges.

How close to the infinite sum can we guarantee that $\sum_{n=2}^{1000}(-1)^n \ln\left(\frac{n+1}{n}\right)$ is?

[FYI: This series, in fact, sums to $\ln\left(\frac{4}{\pi}\right)$.]

2. Use the Taylor series for $f(x) = e^x$, centered at $x = 0$, to find a power series (centered at 0) whose sum is

$$g(x) = \frac{e^x - 1}{x}.$$

Use this to compute $g^{(85)}(0)$ (that is, the 85-th derivative of g, evaluated at $x = 0$).

[Note: As written, $g(x)$ is not defined at $x = 0$. By declaring $g(0) = 1$, we do make it continuous (and differentiable), as your work on this problem will show!]

3. Find the Taylor polynomial $P_4(x)$ of degree 4, centered at $x = 0$, for the function

$$f(x) = x\ln(x+1)$$

Using Taylor's Theorem, give a bound on the size of the error in using $P_4(x)$ to estimate $f(x)$, when $-0.2 < x < 0.2$.

4. For the polar curve

$$r = 1 + 2\sin\theta,$$

find the values of θ, $0 \leq \theta \leq 2\pi$ where the curve has a *horizontal* tangent line. [You may leave your answers in a "pure" form, as values of the functions $\arctan x$, $\arcsin x$, etc.]

5. Find the area inside one petal of the 3-petaled rose, given by the polar equation

$$r = 1 + \sin(3\theta)$$

[One petal is defined by consecutive values of θ for which $r = 0$; you should find such a pair as part of your solution.]

6. Find the length of the polar curve

$$r = 1 - \theta^2$$

from $\theta = -1$ to $\theta = 1$.