

Math 107H Exam 1 Practice Problems

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

Find each of the following integrals.

Note that “ $\int_3^x f(t) dt + C$ ” is not a sufficient computation of an antiderivative!

Some formulas of potential use can be found at the bottom of the last page of the exam.

1. (10 pts.) $\int (x+2)^{3/2} dx$

2. (15 pts.) $\int_0^{\pi/2} \sin^3 x dx$

3. (10 pts.) $\int \frac{x^2 + x - 3}{x^{1/2}} dx$

4. (15 pts.) $\int_0^1 e^{\sqrt{x}} dx$

5. (15 pts.) $\int \frac{dx}{(x+1)^2(x+4)}$

6. (15 pts.) $\int e^{-x} \sin(3x) dx$

7. (20 pts.) $\int (x^2 + 1)^{3/2} dx = \int (\sqrt{x^2 + 1})^3 dx$

Some formulas:

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$c^2 \int \frac{dy}{(y^2 + c^2)^k} = \frac{1}{(2k-2)} \cdot \frac{y}{(y^2 + c^2)^{k-1}} + \frac{(2k-3)}{(2k-2)} \int \frac{dy}{(y^2 + c^2)^{k-1}}$$

1. (10 pts. each) Find the following integrals:

(a): $\int_1^4 x^2 \ln x dx$

(b): $\int \sin^2 x \cos^3 x dx$

2. (10 pts. each) When you apply the appropriate trigonometric substitutions, what do the following integrals become?

(a): $\int \frac{\sqrt{4-x^2}}{x^2} dx$

(b): $\int \frac{x^2}{\sqrt{4x^2+9}} dx$

6. (15 pts.) Recall that if a function f has second derivative satisfying $|f''(x)| \leq M$ for every x in the interval $[a, b]$, then the error E_n in approximating the integral $\int_a^b f(x) dx$ using the trapezoidal rule using n equal subintervals is at most

$$M \frac{(b-a)^3}{12n^2}$$

Based on this, how many subintervals should we divide the interval $[2, 5]$ into in order to be sure to approximate the integral $\int_2^5 x \ln x dx$ with an error of less than $\frac{1}{100}$?