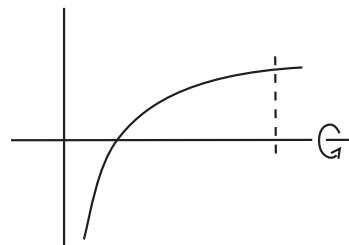


Math 107H Practice Problems for Exam 2

Note: These problem do not quite cover every topic that we have explored; e.g., they do not touch on volume not coming from regions of revolution, work, or compound interest. They should therefore be treated as a “supplement” to your other studies!

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

- (20 pts.) Find the volume of the region obtained by revolving the region under the graph of $f(x) = \ln x$ from $x = 1$ to $x = 3$ around the x -axis (see figure).



- Find the improper integral $\int_2^{\infty} \frac{1}{x(\ln x)^3} dx$.

- Determine the convergence or divergence of the following sequences:

(a) $a_n = \frac{n^3 + 6n^2 \ln n - 1}{2 - 3n^3}$

(b) $b_n = \frac{n^{n+\frac{1}{n}}}{(n+3)^n}$

- Determine the convergence or divergence of the following series:

(a) $\sum_{n=2}^{\infty} \frac{1}{(n-1)(\ln n)^{2/3}}$ [Hint: limit compare, then integral...]

(b) $\sum_{n=0}^{\infty} \frac{6n}{(1-n^2)^2}$

- Set up, **but do not evaluate**, the integral which will compute the arclength of the graph of $y = x\sqrt{1+x^2}$ from $x = 0$ to $x = 3$.

- Find the following limits:

(a) $\lim_{n \rightarrow \infty} \frac{1 + \sqrt{2n}}{\sqrt{n}}$

$$(b) \lim_{n \rightarrow \infty} \frac{4^n + 3^n}{4^n - 3^n}$$

8. Use a comparison test to determine the convergence or divergence of each of the following series:

$$(a) \sum_{n=0}^{\infty} \frac{n^{\frac{1}{3}}}{\sqrt{n^3 + 7}}$$

$$(b) \sum_{n=1}^{\infty} \frac{n + 2^n}{n^2 2^n}$$

3. (15 pts.) Use a comparison theorem to determine whether or not the following improper integral converges:

$$\int_2^{\infty} \frac{\sqrt{1+x+x^3}}{x^2-1} dx$$