Math 107H

Final Exam Practice Problems

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. **Find any THREE (3) of the following FOUR (4) integrals** (5 pts. each)

1-1: $\int \sec^3 x \tan^3 x \; dx$

1-2: $\int \frac{x^2}{\sqrt{3-x^2}} \; dx$

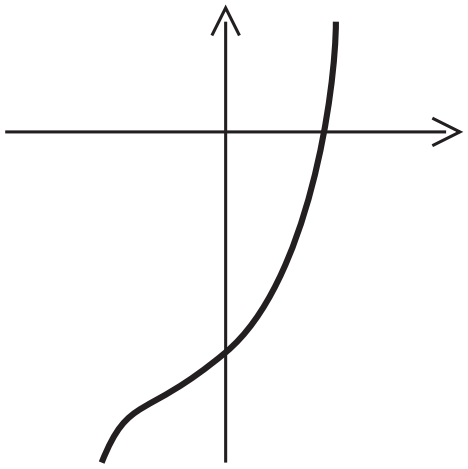
1-3: $\int x^2 e^{3x} \; dx$

1-4: $\int \frac{2x+3}{x^3+x^2-2} \; dx$

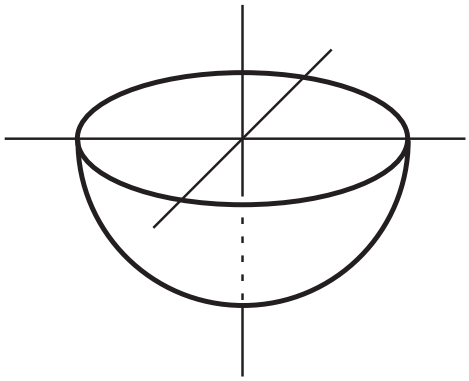
2. (15 pts.) Find the area of the region lying between the graphs of the functions

$$f(x) = 2x - 1 \quad \text{and} \quad g(x) = x^4 + x - 1$$

3. (20 pts.) Find the volume of the region obtained by revolving the region lying between the graph of $f(x) = x^3 + 7x - 22$ and the x- and y-axis around the line $x = -2$.



4. (15 pts.) Find the work done in digging a hemispherical hole in the ground, having a depth of 12 meters. (That is, find the work required to lift the dirt to the surface; assume that dirt has a density of 300 kg per cubic meter.)



5. Find the following limits (10 pts. each):

(a): $\lim_{x\to\infty} \dfrac{x^2 - 3x^3 + 9}{4x^2 - 6x + 1}$

(b): $\lim_{x\to\infty} \dfrac{(x^2+1)^x}{(x+1)^{2x}}$

6. **Determine the convergence or divergence of any THREE (3) of the following FOUR (4) series** (5 pts. each)

6-1 $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^{1/2}}{n^2}$

6-2 $\displaystyle\sum_{n=2}^{\infty} \frac{n!}{(n^2+n-3)^{3/2}}$

6-3 $\displaystyle\sum_{n=0}^{\infty} \Big(\frac{n+3}{3n-5}\Big)^n$

6-4 $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{5/3}}$

7. (20 pts.) Find the degree three Taylor polynomial, centered at $c = 3$, for the function

$$f(x) = (x^2 - 5)^{5/2}$$

8. (20 pts.) Find the arclength of the parametrized curve

$$x = t^4 \quad , \quad y = t^6$$
$$0 \le t \le 2$$