

Quiz number 1 Solutions

Find the following integrals.

$$1. \int_0^2 x^{\frac{3}{2}} - 2x^2 + 7 \, dx$$

$$\text{Since } \int x^{\frac{3}{2}} \, dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{5}x^{\frac{5}{2}} + C, \int 2x^2 \, dx = 2 \int x^2 \, dx = 2 \frac{1}{3}x^3 + C,$$

$$\text{and } \int 7 \, dx = 7x + c, \text{ we have}$$

$$\begin{aligned} \int_0^2 x^{\frac{3}{2}} - 2x^2 + 7 \, dx &= \left. \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^3 + 7x \right|_0^2 \\ &= \left[\frac{2}{5}2^{\frac{5}{2}} - \frac{2}{3}2^3 + 7 \cdot 2 \right] - \left[\frac{2}{5}0^{\frac{5}{2}} - \frac{2}{3}0^3 + 7 \cdot 0 \right] \\ &= \frac{2}{5}2^{\frac{5}{2}} - \frac{2}{3}2^3 + 7 \cdot 2 \text{ (which is a perfectly fine number...)} \\ &= \frac{8}{5}\sqrt{2} - \frac{16}{3} + 14 = \frac{8}{5}\sqrt{2} + \frac{26}{3} . \end{aligned}$$

$$2. \int (3x + 1)^{2/3} \, dx$$

By making the u -substitution $u = 3x + 1$, with $du = 3 \, dx$, we have

$$\begin{aligned} &\int (3x + 1)^{2/3} \, dx \\ &= \int \frac{1}{3}(3x + 1)^{2/3}(3 \, dx) \\ &= \frac{1}{3} \int (3x + 1)^{2/3}(3 \, dx) \\ &= \frac{1}{3} \int (u)^{2/3} \, du \Big|_{u=3x+1} \\ &= \frac{1}{3} \left[\frac{3}{5}(u)^{5/3} \right] + C \Big|_{u=3x+1} \\ &= \frac{1}{5}(3x + 1)^{5/3} + C . \end{aligned}$$

[We can check our work: differentiate our answer!]