Math 107H, Section 1

Quiz number 2 solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

Find the following antiderivative:

$$\int \frac{\ln x}{x^2}\ dx$$

There are (at least) two ways to proceed:

By parts:

Setting $u = \ln x$ and $dv = \frac{dx}{x^2} = x^{-2}\ dx$, we have $du = \frac{dx}{x}$ and $v = -x^{-1}$, so

$$\int \frac{\ln x}{x^2}\ dx = -x^{-1}\ln x - \int \frac{-x^{-1}}{x}\ dx = -x^{-1}\ln x + \int x^{-2}\ dx = -x^{-1}\ln x - x^{-1} + C$$

.

Or:

By u-substitution:

Setting $u = \ln x$, we have $du = \frac{dx}{x}$ and $x = e^u$, so

$$\int \frac{\ln x}{x^2}\ dx = \int \frac{\ln x}{x}\frac{dx}{x} = \int \frac{u}{e^u}\ du\ \Big|_{u=\ln x} = \int ue^{-u}\ du\ \Big|_{u=\ln x},$$

and we continue by parts(!).

Setting $v = u$ and $dw = e^{-u}\ du$, we have $dv =\ du$ and $w = -e^{-u}$, so

$$\int ue^{-u}\ du = -ue^{-u} - \int -e^{-u}\ du = -ue^{-u} + \int e^{-u}\ du = -ue^{-u} - e^{-u} + C.$$

So

$$\int \frac{\ln x}{x^2}\ dx = \int ue^{-u}\ du\ \Big|_{u=\ln x} = -ue^{-u} - e^{-u} + C\ \Big|_{u=\ln x}$$

$$= -(\ln x)e^{-\ln x} - e^{-\ln x} + C = -x^{-1}\ln x - x^{-1} + C\ .$$