Math 107H, Section 1

Quiz number 4 solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

5. Find the following integral:

$$\int \frac{x^2\ dx}{(x-2)(x+1)^2}\ .$$

We use partial fractions to express $\dfrac{x^2\ dx}{(x-2)(x+1)^2}$ in a more reasonable way:

Since the denominator factors into linears (one of which appears twice), we know that

$$\begin{aligned}\frac{x^2\ dx}{(x-2)(x+1)^2} &= \frac{A}{x-2}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\\ &= \frac{A(x+1)^2+B(x-2)(x+1)+C(x-2)}{(x-2)(x+1)^2}\ .\end{aligned}$$

so we need to solve $x^2 = A(x+1)^2+B(x-2)(x+1)+C(x-2)$ for A, B, and C.

Setting $x=-1$, we have $1=(-1)^2=A(0)+B(-3)(0)+C(-3)=-3C$, so $C=\dfrac{-1}{3}$.

Setting $x=2$, we have $4=(2)^2=A(3)^2+B(0)(3)+C(0)=9A$, so $A=\dfrac{4}{9}$.

Finally, (picking any other number...) setting $x=4$ (just for fun), since we know the values of A and C we have

$$16=(4)^2=\frac{4}{9}(5)^2+B(2)(5)-\frac{1}{3}(2)=10B+\frac{100}{9}-\frac{6}{9}=\frac{94}{9}\ .$$

So $10B=16-\dfrac{94}{9}=\dfrac{144-94}{9}=\dfrac{50}{9}$, so $B=\dfrac{5}{9}$

$$\begin{aligned}\underline{\text{So}},\ \int \frac{x^2\ dx}{(x-2)(x+1)^2} &= \int \frac{4}{9}(x-2)^{-1}+\frac{5}{9}(x+1)^{-1}-\frac{1}{3}(x+1)^{-2}\ \text{dx}\\ &= \frac{4}{9}\ln|x-2|+\frac{5}{9}\ln|x+1|-\frac{1}{3}\frac{(x+1)^{-1}}{-1}+C\\ &= \frac{4}{9}\ln|x-2|+\frac{5}{9}\ln|x+1|+\frac{1}{3}(x+1)^{-1}+C\ .\end{aligned}$$