

Quiz number 5 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

5. Find the following improper integral:

$$\int_2^{\infty} \frac{dx}{x(\ln x)^3}.$$

We need an antiderivative: $\int \frac{dx}{x(\ln x)^3} = \int \frac{1}{(\ln x)^3} \frac{1}{x} dx$

Set $u = \ln x$, so $du = \frac{1}{x} dx$, then

$$\int \frac{dx}{x(\ln x)^3} = \int \frac{du}{u^3} \Big|_{u=\ln x} = \int u^{-3} du \Big|_{u=\ln x} = \frac{u^{-2}}{-2} \Big|_{u=\ln x} = \frac{-1}{2}(\ln x)^{-2}.$$

So

$$\begin{aligned} \int_2^{\infty} \frac{dx}{x(\ln x)^3} &= \lim_{N \rightarrow \infty} \int_2^N \frac{dx}{x(\ln x)^3} = \lim_{N \rightarrow \infty} \frac{-1}{2}(\ln x)^{-2} \Big|_2^N \\ &= \lim_{N \rightarrow \infty} \frac{-1}{2(\ln N)^2} - \frac{-1}{2(\ln 2)^2} \end{aligned}$$

Since $\ln N \rightarrow \infty$ as $N \rightarrow \infty$, $\frac{-1}{2(\ln N)^2} \rightarrow 0$ as $N \rightarrow \infty$, and so

$$\int_2^{\infty} \frac{dx}{x(\ln x)^3} = 0 - \frac{-1}{2(\ln 2)^2} = \frac{1}{2(\ln 2)^2}.$$