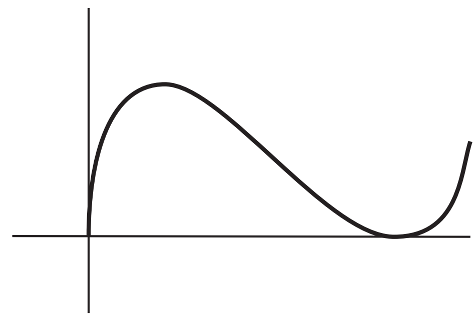
Math 107H, Section 1

Quiz number 6 Solution

Show all work! How you get your answer is just as important, if not more important, than the answer itself.

Find the volume of the "turnip" obtained by revolving the region lying between the graph of the function $f(x) = (x-2)^2\sqrt{x}$ and the x-axis around the x-axis (see figure).



Rotating the vertical line from $y = 0$ to $y = (x-2)^2\sqrt{x}$ around the x-axis yields a disk with radius $r = (x-2)^2\sqrt{x}$, and so has area

$$A(x) = \pi r^2 = \pi[(x-2)^2\sqrt{x}]^2 = \pi(x-2)^4 x$$

Since the vertical lines that hit the region pictured run between the two values of x where $(x-2)^2\sqrt{x} = 0$, namely $x = 0$ and $x = 2$, we have

$$\text{Volume} = \int_0^2 A(x)\ dx = \pi \int_0^2 x(x-2)^4\ dx$$

We can either multiply $(x-2)^4$ out, to integrate the resulting polynomial, or use u-substitution:

Setting $u = x - 2$, so $du = dx$, $x = u + 2$, and $x = 0 \Rightarrow u = -2$, $x = 2 \Rightarrow u = 0$, we have

$$\int_0^2 x(x-2)^4\ dx = \int_{-2}^0 (u+2)u^4\ du = \int_{-2}^0 u^5 + 2u^4\ du = \tfrac{1}{6}u^6 + \tfrac{2}{5}u^5 \Big|_{-2}^{0}$$

$$= [\frac{1}{6}0^6 + \frac{2}{5}0^5] - [\frac{1}{6}(-2)^6 + \frac{2}{5}(-2)^5] = -\frac{2^6}{6} + \frac{2^6}{5} = 64\frac{-5+6}{6\cdot 5} = \frac{64}{30} = \frac{32}{15}$$