Math 107H, Section 1

Quiz number 7 Solutions

Show all work! How you get your answer is just as important, if not more important, than the answer itself.

Determine the limits of the following sequences (if they exist):

(a) $a_n = \dfrac{3n^2+n-3}{17n^2-7n-2}$

Since $a_n = \dfrac{3n^2+n-3}{17n^2-7n-2} == \dfrac{1/n^2}{1/n^2}\dfrac{3n^2+n-3}{17n^2-7n-2} = \dfrac{3+\frac{1}{n}-3(\frac{1}{n})^2}{17-7\frac{1}{n}-2(\frac{1}{n})^2}$

and $\dfrac{1}{n} \to 0$ as $n \to \infty$, we have

$a_n \to \dfrac{3+(0)-3(0)^2}{17-7(0)-2(0)^2} = \dfrac{3}{17} \qquad$ as $n \to \infty$.

(b) $b_n = \sqrt{n^2+2} - \sqrt{n^2-2}$ [Hint: multiply by a useful version of 1 ...!]

If we multiply by $\dfrac{\sqrt{n^2+2}+\sqrt{n^2-2}}{\sqrt{n^2+2}+\sqrt{n^2-2}}$ we can make the numerator more manageable:

$$b_n = \sqrt{n^2+2} - \sqrt{n^2-2} = (\sqrt{n^2+2} - \sqrt{n^2-2})\frac{\sqrt{n^2+2}+\sqrt{n^2-2}}{\sqrt{n^2+2}+\sqrt{n^2-2}}$$

$$= \frac{[\sqrt{n^2+2}]^2 - [\sqrt{n^2-2}]^2}{\sqrt{n^2+2}+\sqrt{n^2-2}} = \frac{(n^2+2)-(n^2-2)}{\sqrt{n^2}\sqrt{1+2(\frac{1}{n})^2}+\sqrt{n^2}\sqrt{1-2(\frac{1}{n})^2}}$$

$$= \frac{4}{n\left(\sqrt{1+2(\frac{1}{n})^2}+\sqrt{1-2(\frac{1}{n})^2}\right)}$$

Since $\sqrt{1+2(\dfrac{1}{n})^2}$ and $\sqrt{1-2(\dfrac{1}{n})^2} \to \sqrt{1+0} = 1$ as $n \to \infty$, we have

$$b_n = 4(\frac{1}{n})\frac{1}{\sqrt{1+2(\frac{1}{n})^2}+\sqrt{1-2(\frac{1}{n})^2}} \to (4)(0)(\frac{1}{1+1}) = 0 \text{ as } n \to \infty \text{ .}$$