Math 107H, Section 1

Quiz number 8 Solutions

Show all work! How you get your answer is just as important, if not more important, than the answer itself.

Determine whether or not the following series converge:
[Note: your work from one will help you with the other!]

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^4 5^n}{n!}$

Setting $a_n = \dfrac{n^4 5^n}{n!}$, we have

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^4 5^{n+1}}{(n+1)!} \Big/ \frac{n^4 5^n}{n!}\right| = \frac{(n+1)^4}{n^4} \cdot \frac{5^{n+1}}{5^n} \cdot \frac{n!}{(n+1)!}$$

$$= \left(\frac{n+1}{n}\right)^4 \cdot 5 \cdot \frac{n!}{n!(n+1)} = \left(1 + \frac{1}{n}\right)^4 \cdot 5 \cdot \frac{1}{n+1}.$$

But since as $n \to \infty$ we have $\dfrac{1}{n} \to 0$ and $\dfrac{1}{n+1} \to 0$, then

$\left|\dfrac{a_{n+1}}{a_n}\right| \to (1+0)^4 \cdot 5 \cdot 0 = 0 < 1$, and so by the Ratio(n) Test, $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^4 5^n}$

Setting $b_n = \dfrac{n!}{n^4 5^n}$, we note that $b_n = \dfrac{1}{a_n}$, and so
$\dfrac{b_{n+1}}{b_n} = \dfrac{a_n}{a_{n+1}}$. So since $\dfrac{a_{n+1}}{a_n} \to 0$ as $n \to \infty$, we know that $\dfrac{b_{n+1}}{b_n} \to \infty$ as $n \to \infty$.
In particular, the terms b_n blow up, and so $\displaystyle\sum_{n=0}^{\infty} b_n$ diverges, by the n-th term test. [Or, if you prefer, since $\infty > 1$ (essentially), it diverges by the Ratio(n) Test.]