Math 107H

A not-so-short table of integrals

First building blocks:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \text{ (provided } n \neq -1) \qquad \int \frac{1}{x} \, dx = \ln|x| + C \qquad \int e^{ax} \, dx = \frac{e^{ax}}{a} + C$$

$$\int \sin(kx) \, dx = \frac{-\cos(kx)}{k} + C \qquad \int \cos(kx) \, dx = \frac{\sin(kx)}{k} + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C \qquad \int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C \qquad \int \csc x \, dx = -\ln|\csc x + \cot x| + C \qquad \int \csc x \cot x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \cot x \, dx = \ln|\sin x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \text{Arcsin}(\tfrac{x}{a}) + c \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a} \text{ Arctan}(\tfrac{x}{a}) + c \qquad \int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\text{Arcsec}(\tfrac{x}{a}) + c$$

Reduction formulas:

$$\int x^n e^{ax} \, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx \qquad \text{[by parts: } u = x^n]$$

$$\int x^n \sin(bx) \, dx = \frac{-1}{b} x^n \cos(bx) + \frac{n}{b} \int x^{n-1} \cos(bx) \, dx \qquad \text{[by parts: } u = x^n]$$

$$\int x^n \cos(bx) \, dx = \frac{1}{b} x^n \sin(bx) - \frac{n}{b} \int x^{n-1} \sin(bx) \, dx \qquad \text{[by parts: } u = x^n]$$

$$n \int \sin^n x \, dx = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \, dx \qquad \text{[by parts: } dv = \sin x \, dx]$$

$$n \int \cos^n x \, dx = \cos^{n-1} x \sin x - (n-1) \int \cos^{n-2} x \, dx \qquad \text{[by parts: } dv = \cos x \, dx]$$

$$(n-1) \int \sec^n x \, dx = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x \, dx \qquad \text{[by parts: } dv = \sec^2 x \, dx]$$

$$\int \tan^n x \, dx = [1/(n-1)] \tan^{n-1} x - \int \tan^{n-2} x \, dx \qquad \text{[by } u\text{-subs: } \tan^2 x = \sec^2 x - 1]$$

$$\int \cot^n x \, dx = -\int \tan^n u \, du \,\Big|_{u=\frac{\pi}{2}-x} \qquad \int \csc^n x \, dx = -\int \sec^n u \, du \,\Big|_{u=\frac{\pi}{2}-x}$$

Special forms:

$$\int e^{ax} \sin bx \, dx = \frac{1}{a^2+b^2}[ae^{ax}\sin(bx) - be^{ax}\cos(bx)] + C \text{ [Or: remember the basic form, and } \tfrac{d}{dx} \text{ !]}$$

$$\int e^{ax} \cos bx \, dx = \frac{1}{a^2+b^2}[be^{ax}\sin(bx) + ae^{ax}\cos(bx)] + C \text{ [Or: remember the basic form, and } \tfrac{d}{dx} \text{ !]}$$

$\sin(ax)\sin(bx) = (1/2)[\cos(a-b)x) - \cos((a+b)x)]$; integrate <u>that</u> instead! [Or: by parts, twice!]

$\sin(ax)\cos(bx) = (1/2)[\sin(a+b)x) + \sin((a-b)x)]$; integrate <u>that</u> instead! [Or: by parts, twice!]

$\cos(ax)\cos(bx) = (1/2)[\cos(a-b)x) + \cos((a+b)x)]$; integrate <u>that</u> instead! [Or: by parts, twice!]

$\int \sin^n x \cos^m x \, dx$:

- if n or m odd, set aside one of $\sin x \, dx$ or $\cos x \, dx$ and convert the rest, using $\sin^2 x + \cos^2 x = 1$
- if both n, m even, convert all to powers of $\sin x$ and use reduction formula

$$\int \sec^n x \tan^m x \, dx = \int \frac{\sin^m x}{\cos^{n+m} x} \, dx :$$

- if n even, keep two, convert using $\sec^2 x = \tan^2 x + 1$, and u-sub: integrand is (powers of $\tan x$)($\sec^2 x \, dx$)
- if m odd, keep one each, using $\tan^2 x = \sec^2 x - 1$, and u-subs: integrand is (powers of $\sec x$)($\sec x \tan x \, dx$)
- otherwise, convert $\tan x$'s to $\sec x$'s and use reduction formula.

$$\int \frac{\cos^m x}{\sin^{n+m} x} \, dx = \int \csc^n x \cot^m x \, dx = -\int \sec^n u \tan^m u \, du \,\Big|_{u=\frac{\pi}{2}-x}$$

$$\int \frac{dx}{\sin^n x \cos^m x} = \int \frac{\sin x \, dx}{\sin^{n+1} x \cos^m x} = \int \frac{\cos x \, dx}{\sin^n x \cos^{m+1} x} \; ; \qquad n, m \text{ both even is a \underline{bear}...}$$

$$\frac{1}{\sin^{2n} x \cos^{2m} x} = \frac{\cos^{2k} x \text{ or } \sin^{2k} x}{(\sin x \cos x)^{2r}} = \frac{[\frac{1}{2}(1 \pm \cos(2x)]^k}{[\frac{1}{2}\sin(2x)]^r}; \; u\text{-sub } (u = 2x) \text{ and expand out... !}$$