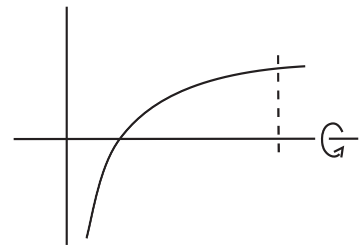
Math 107H Practice Problems for Exam 2

Note: These problems do not quite cover every topic that we have explored; e.g., they do not touch on volume not coming from regions of revolution, work, or compound interest. They should therefore be treated as a "supplement" to your other studies!

Show all work. How you get your answer is just as important, if not more important, than the answer itself.



1. Find the volume of the region obtained by revolving the region under the graph of $f(x) = \ln x$ from $x = 1$ to $x = 3$ around the x-axis (see figure).

2. Find the improper integral $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^3}\,dx$.

3. Determine the convergence or divergence of the following sequences:

(a) $a_n = \dfrac{n^3 + 6n^2 \ln n - 1}{2 - 3n^3}$ (b) $b_n = \dfrac{n^{n+\frac{1}{n}}}{(n+3)^n}$

4. *Use the integral test* to determine the convergence or divergence of the following series:

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(n)(\ln n)^{2/3}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{6n}{(1-n^2)^2}$

6. Set up, **but do not evaluate**, the integral which will compute the arclength of the graph of $y = x\sqrt{1+x^2}$ from $x = 0$ to $x = 3$.

6. Fin the following limits:

(a) $\displaystyle\lim_{n\to\infty} \frac{1+\sqrt{2n}}{\sqrt{n}}$ (b) $\displaystyle\lim_{n\to\infty} \frac{4^n + 3^n}{4^n - 3^n}$

1. Use a comparision theorem to decide if the following improper integral converges (if yes, you do *not* need to find the value of the integral):

$$\int_7^\infty \frac{x \ln x}{x^2+1}\,dx$$

2. Find the volume of the region obtained by spinning the triangle with sides lying along the lines $y = \frac{1}{2}x$, $x = 4$, and $y = 1$, *around the line* $y = -2$.

3. Set up, *but do not evaluate*, the integral which evaluates to the length of the spiral, with parametric equation

$$x = t\cos t\ ,\ y = t\sin t\ , \qquad \text{for } 0 \le t \le 4\pi\ .$$

4. Find the limit of each of the following sequences, *if it exists*:

(a) $a_n = \dfrac{2 + \sqrt{n^2 + 5n - 1}}{7n + 12}$

(b) $b_n = (n^2 + 2)^{\frac{1}{n}}$ [Hint: take logs, first!]

[Also, recall *L'Hopital's Rule*: if $f(x), g(x) \to \infty$ as $x \to \infty$, then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)} \text{ .]}$$