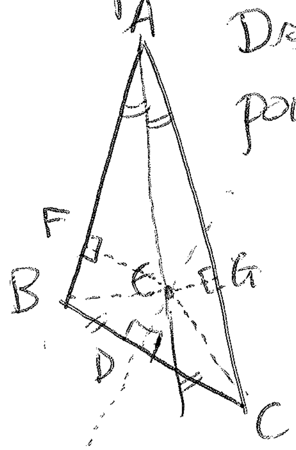
George Polya: "Geometry is the science of correct reasoning on incorrect figures."

As an illustration, we have: <u>Theorem</u>: <u>All</u> <u>triangles</u> <u>are</u> <u>"isosceles"</u>.
This is of course false! But with the <u>wrong</u> figure, we can <u>"prove"</u> it!

Given $\triangle ABC$, if the angle $\measuredangle BAC$ meets side BC perpendicularly (at D) then $\measuredangle BDA = \measuredangle CDA = \frac{\pi}{2}$, $\measuredangle BAD = \measuredangle CAD$ (bisector), and $AD = AD$ means $\triangle BDA \cong \triangle CDA$ (angle-side-angle), so $BA = CA$ and $\triangle ABC$ <u>is</u> isosceles.

On the other hand, if the angle bisector does <u>not</u> meet side BC at right angles, then this line and the perpendicular bisector of BC (at midpoint D) will <u>meet</u>, at a point E (see figure). <u>Then</u>:



Drop perpendiculars from E to sides AB and AC, to points F and G. Note that:

$\measuredangle EDB = \measuredangle EDC = \frac{\pi}{2}$ and $BD = CD$, and $ED = ED$, so $\triangle EDB \cong \triangle EDC$ (side-angle-side) and so $BE = CE$.

Also, $\measuredangle FAE = \measuredangle GAE$ (angle bisector), and $\measuredangle FEA = \measuredangle GEA$ (because of this and other angles are $\frac{\pi}{2}$), and $AE = AE$, so $\triangle AEF \cong \triangle AEG$ (angle-side-angle), so $\boxed{AF = AG}$, $FE = GE$, and <u>and</u> $FE = GE$. Finally, $\measuredangle BFE = \measuredangle CGE = \frac{\pi}{2}$, $BE = CE$, so $\triangle BFE \cong \triangle CGE$ (angle-side-side for <u>right</u> triangles, which is <u>true</u>!). So $\boxed{BF = CG}$. <u>So</u>

$AB = AF + FB = AG + GC = AC$, and $\triangle ABC$ is <u>isosceles</u>! ///

<u>The problem</u>: For any <u>actual</u>, non-isosceles triangle, the point E will <u>always</u> be <u>outside</u> of the triangle! The "figure" above is wrong!