Math 107H, Section 1

Quiz number 4 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

Find the following antiderivatives:

5. $\int x^2 e^{2x}\ dx$

Using the u-substitution $u = 2x$, we get

$$\int e^{2x}\ dx = \int e^u \frac{1}{2}\ du\Big|_{u=2x} = \frac{1}{2}e^u + C\Big|_{u=2x} = \frac{1}{2}e^{2x} + C. \text{ Then:}$$

$u = x^2$ and $dv = e^{2x}\ dx$ gives $du = 2x\ dx$ and $v = \frac{1}{2}e^{2x}$, so

$$\int x^2 e^{2x}\ dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}\cdot 2\int xe^{2x}\ dx = \frac{1}{2}x^2 e^{2x} - \int xe^{2x}\ dx$$

Then $u = x$ and $dv = e^{2x}\ dx$ gives $du = dx$ and $v = \frac{1}{2}e^{2x}$, so

$$\int xe^{2x}\ dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\ dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C.$$

Putting these together yields

$$\int x^2 e^{2x}\ dx = \frac{1}{2}x^2 e^{2x} - (\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}) + C = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

6. $\int x^3 \ln x\ dx$

Since $\ln x$ is more troublesome to integrate than e^{2x}, we go the opposite route:

Setting $u = \ln x$ and $dv = x^3\ dx$ gives $du = \frac{1}{x}\ dx$ and $v = \frac{1}{4}x^4$, so

$$\int x^3 \ln x\ dx = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int \frac{1}{x}x^4\ dx$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\ dx = \frac{1}{4}x^4 \ln x - \frac{1}{4}(\frac{1}{4}x^4) + C = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$$

N.B.: We can alway check our work by differentiating!

$$\frac{d}{dx}(\frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C) = \frac{1}{2}(2xe^{2x} + x^2(2e^{2x})) - \frac{1}{2}(e^{2x} + x(2e^{2x}) + \frac{1}{4}(2e^{2x}) + 0$$

$$= xe^{2x} + x^2 e^{2x} - \frac{1}{2}e^{2x} - xe2x + \frac{1}{2}e^{2x} = x^2 e^{2x} + xe^{2x} - xe^{2x} - \frac{1}{2}e^{2x} + \frac{1}{2}e^{2x}$$

$$= x^2 e^{2x}\ .$$

$$\frac{d}{dx}(\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C) = \frac{1}{4}(4x^3 \ln x + x^4 \frac{1}{x}) - \frac{1}{16}(4x^3) + 0$$

$$= x^3 \ln x + \frac{1}{4}x^3 - \frac{1}{4}x^3 = x^3 \ln x\ .$$