Math 107H, Section 1

Quiz number 6 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

8. Find the following improper integral:

$$\int_2^\infty \frac{dx}{x(\ln x)^4}\ .$$

Since it says to find the integral, we don't want to compare, we want to compute!

$$\int_2^\infty \frac{dx}{x(\ln x)^4} = \lim_{N\to\infty}\int_2^N \frac{dx}{x(\ln x)^4},\text{ so we first find } \int \frac{dx}{x(\ln x)^4}\ :$$

Using u-substitution, $u = \ln x$ gives $du = \frac{1}{x}\ dx$, so

$$\int \frac{dx}{x(\ln x)^4} = \int \frac{1}{(\ln x)^4}\frac{1}{x}\ dx = \int \frac{1}{u^4}\ du\Big|_{u=\ln x} = \int u^{-4}\ du\Big|_{u=\ln x}$$

$$= \frac{u^{-3}}{-3}\Big|_{u=\ln x} = \frac{-1}{3(\ln x)^3)} + C.$$

So:

$$\int_2^\infty \frac{dx}{x(\ln x)^4} = \lim_{N\to\infty}\frac{-1}{3(\ln x)^3)}\Big|_2^N = \lim_{N\to\infty}\left(\frac{-1}{3(\ln N)^3)} - \frac{-1}{3(\ln 2)^3)}\right)$$

$$= \lim_{N\to\infty}\frac{1}{3}((\ln 2)^{-3} - (\ln N)^{-3})$$

But since $\ln N \to \infty$ as $N \to \infty$, $(\ln N)^{-3} \to 0$ as $N \to \infty$, so

$$\int_2^\infty \frac{dx}{x(\ln x)^4} = \lim_{N\to\infty}\left(\frac{1}{3}((\ln 2)^{-3} - (\ln N)^{-3})\right) = \frac{1}{3}((\ln 2)^{-3} - 0) = \frac{1}{3}(\ln 2)^{-3}\ .$$