FINAL EXAM

Math 107, Spring Semester 2008

Name (Print):______________________________

Student ID Numer:______________________________

TA Name:______________________________

Please Circle your professor's name:
M. Foss M. Rammaha B. Rogge R. Wiegand

Please Circle your class time:
8:30 M,W,F 9:30 T,R 10:30 M,W,F 12:30 M,W,F 6:30 M,W

INSTRUCTIONS:

- *Be sure to write your name on each page. There are 8 pages of questions and this cover sheet.*
- *SHOW ALL YOUR WORK. Partial credit will be given only if your work is relevant and correct.*
- *Please make your work as clear and easy to follow as possible.*
- *Do not spend too long on any one problem—note the point value of the problem when deciding how much time to spend! You do not need to work the problems in the order in which they appear.*
- *No books or notes are allowed. Calculators and electronic devices are* **NOT permitted.**

Question	Points	Score
1	24	
2	12	
3	18	
4	8	
5	12	
6	12	
7	16	
8	12	
9	24	
10	16	
11	20	
12	26	
Total	200	

1. [24 Points] Evaluate the following integrals:

a) [12 Points] $\displaystyle\int \frac{9x-3}{(2x-1)(x+1)}\,dx$

b) [12 Points] $\displaystyle\int x^4 \ln x\,dx$

2. [12 Points] For what values of x does the following series converge? For these values of x, what is the sum of the series?

$$\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{3}{5}x\right)^k = \frac{3}{5}x - \left(\frac{3}{5}x\right)^2 + \left(\frac{3}{5}x\right)^3 - \cdots \tag{1}$$

3. [18 Points] Determine whether the following improper integrals are convergent or divergent. If the integral is convergent, find its exact value.

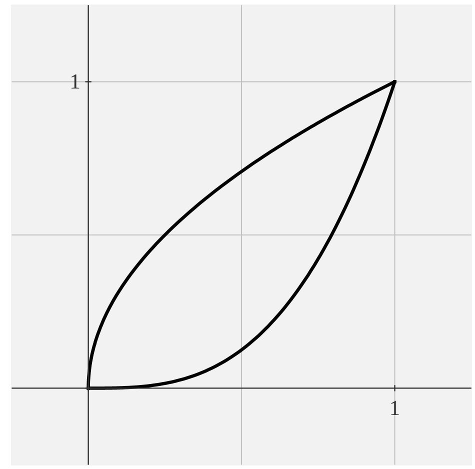
a) [9 Points] $\displaystyle\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

b) [9 Points] $\displaystyle\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

4. [8 Points] By using a suitable comparison Theorem, determine whether the following improper integral is convergent or divergent:

$$\int_1^\infty \frac{x^2}{4x^3-x+1}\,dx$$

5. [12 Points] Let **R** be the bounded region in the first quadrant enclosed by the graphs of $y = x^3$ and $y = \sqrt{x}$, as shown. Let **S** be the solid obtained by revolving **R** about the **y-axis**. Find **(but don't evaluate)** an integral whose value gives the volume of **S**. You may use any method of your choice.



6. [12 Points] A tank has the shape of a right circular cone with its vertex on the ground. The height of the tank is 10 feet; the radius of its top is 6 feet. Assume that the tank is filled with oil weighing 50 lb/ft^3. Write down **but do not evaluate** an integral whose value is the work required to pump all of the oil over the top of the tank.

7. [16 Points] Determine whether the following series converge absolutely, converge conditionally or diverge. You must show all details to receive credit.

a) [6 Points] $\displaystyle\sum_{k=1}^{\infty} \frac{k}{2k+3}$

b) [10 Points] $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\sqrt{k}}{3k^2+k-1}$

8. [12 Points] A particle is traveling in space from the point $P = (2, 4, -2)$ to the point $Q = (6, 2, 2)$ on a line segment with speed $2\,cm/min$, where the xyz-coordinate system is measured in centimeters.

a) [6 Points] Find the velocity vector of the particle and total time needed for the trip.

b) [6 Points] Find the parametric equations for the path of the particle.

9. [24 Points] Find the radius of convergence and the largest interval on which the following power series converges absolutely. At each endpoint of the interval, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k} (x-2)^k \tag{2}$$

10. [16 Points] The Taylor series of e^{-x^2} about $x = 0$ is:

$$e^{-x^2} = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!} \, x^{2k}, \quad x \in \mathbb{R}. \tag{3}$$

a) [6 Points] Find a series whose sum is $\displaystyle\int_0^1 e^{-x^2} dx$.

b) [10 Points] Use the series you found in part a) to approximate integral $\displaystyle\int_0^1 e^{-x^2} dx$ with an error that does not exceed 0.03.

11. [20 Points] Consider the vectors $\overrightarrow{v} = \langle 3, -1, -1 \rangle = 3\overrightarrow{i} - \overrightarrow{j} - \overrightarrow{k}$ and $\overrightarrow{w} = \langle 1, 4, -1 \rangle = \overrightarrow{i} + 4\overrightarrow{j} - \overrightarrow{k}$.

a) [5 Points] Find the vector $\overrightarrow{v} + 2\overrightarrow{w}$.

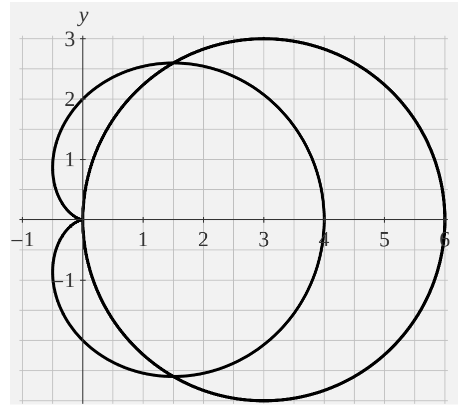
b) [5 Points] Find the cosine of the angle between $\overrightarrow{v}$ and $\overrightarrow{w}$.

c) [5 Points] Find $\text{proj}_{\overrightarrow{w}}\overrightarrow{v}$, i.e., the vector projection of $\overrightarrow{v}$ onto $\overrightarrow{w}$.

d) [5 Points] Does there exist a **unit** vector $\overrightarrow{u}$ that is parallel to the vector $\overrightarrow{w}$ and orthogonal to $\overrightarrow{v}$? If "yes", find it; and if "no" explain why not.

12. [26 Points] The graphs of the following polar curves are as shown:

$$\mathcal{C}_1 : \quad r = 2(1 + \cos\theta); \qquad \mathcal{C}_2 : \quad r = 6\cos\theta.$$



a) [10 Points] Find, but don't evaluate, an integral whose value is the area of the region that lies outside $\mathcal{C}_1$ and inside $\mathcal{C}_2$. (You may use any available symmetry).

b) [8 Points] Find the slope of $\mathcal{C}_2$ at the point that corresponds to $\theta = \frac{\pi}{3}$.

c) [8 Points] Find, but don't evaluate, an integral whose value is the arc length of $\mathcal{C}_1$.