Joy of Numbers – Take Home Test # 2
Due: Thursday, September 29
No Collaboration Allowed

On this test, your work is to be your own with no consultation with any other person (in this class or not) except for the instructor. Feel free to ask me any questions. I won't give you any answers to the problems but will be happy to try to clarify any confusion you may have, probably by asking you more questions. You may feel free to use any written references or books or even the internet, just no consultation with any persons.

As usual, you will be graded both on mathematical content and on clarity of expression. Each problem is worth a maximum of 12 points. Some of the questions are a bit open-ended, so be creative, make conjectures, and back up your assertions with a proof or a counterexample. In writing your answers, use complete English sentences and be sure to say exactly what you mean. Papers will be graded on the basis of what you have written, so be sure to take the time to express yourself clearly. If you are stuck on a problem and have no idea where to begin, a good way to get started is to look at lots of numerical examples and try to find a pattern.

1. Find the greatest common divisor of the numbers $a = 1223456$ and $m = 4433221$. [You should feel free to use mechanical means to do your calculations, but write out your steps in full!]

2. We've seen how to use gcd computations to find integer solutions to linear equations
$$ax + by = c$$
(where a, b, and c are given constants and x and y are variables). Use that approach to find integers x and y so that $117x + 456y = 33$, or show that there can be no solution!

3. List all of the positive divisors of each of the integers, in turn, from 1 to 30. For example, 12 has the six divisors 1, 2, 3, 4, 6, and 12. You will notice that "most" positive integers have an even number of divisors, but a few have an odd number of divisors. Classify those positive integers which have an even number of divisors and those which have an odd number of divisors. Try to explain why your classification works in general. [Hint: one explanation has to do with our observation from class that factors of a number come in 'pairs'...!]

4. For a a positive integer, describe all of the values of $\gcd(a, a + 6)$ that can possibly occur. Why do you know that those are the only values that can occur? For each of those possible values, find two examples of an a that gives that value!