Math 189H Joy of Numbers Activity Log

Tuesday, August 23, 2011

Jim Horning: "Good judgement comes from experience. Experience comes from bad judgement."

Kurt Vonnegut: "A lack of seriousness has led to all sorts of wonderful insights."

Today in our first meeting we spent most of our time discussing the goals of the class and how we will carry out those goals, embodied in our course policy sheet. Those goals are essentially to experience the construction of a mathematical theory, using "stalking the big primes" as our theme. We will look for patterns, formulate conjectures based on the patterns we observe, and try to verify or refute those conjectures using further observation, deduction and reasoning. Our chosen subject matter to practice upon is what most people would call "number theory": the integers and their properties.

Despite an excellent effort (at your instructor's suggestion) to divert the discussion away from mathematics, your instructor managed to insert a few mathematical/number theoretic facts:

The instructor is the proud discoverer of what is at the time of this writing the 98th largest known prime number: it is

$$428551 \cdot 2^{2006520} + 1$$

and has 604,029 digits. This is actually far smaller than the current record holder,

$$2^{43112609} - 1$$

which has an astonishing 12,978,189 digits. (More information on prime number records can be found at http://primes.utm.edu/ .) It was quite naturally asked why one would want to know prime numbers of such size, to which the most reasonable answer is "mostly we don't"; the important point for our ongoing discussion is that it is possible to know that numbers of this size are prime! And on a more pragmatic note, it is in fact the case that a large fraction of the electronic security measures that we rely on for things such as ecommerce (a single company, eServGlobal, reports that it processes 20 million such transactions each day!) base their security in part on the use of largish prime numbers, on the order of hundreds of digits in size. This would not be possible except that in practice we can generate huge numbers of primes of this size [or rather, what are usually called 'industrial-grade primes'; more on this later!] with very little effort (actually, computers do the heavy lifting, but very quickly). Understanding how and why prime numbers play a role in things like ecommerce is one of the ultimate goals for this class.

To stimulate discussion for Thursday, we left with the following questions/tasks:

Find as many ways as you can to describe:

(a) A number n is even. (b) A number n is odd.

How would you convince, based on your description(s) that

even plus even is even even plus odd is odd

odd plus odd is even even times even is even

[fill in the rest!]