Exam 3 Practice problems

3. (15 pts.) Find the arclength of the path

$$\mathbf{c}(t) = (\frac{t^2}{2}, \frac{t^3}{3})$$

from $t = 1$ to $t = 3$

(Hint: once you have your differential of arc length, **factor it**.).

3. (20 pts.) Find the curl of the vector field

$$F(x, y, z) = (2xyz, x^2 - xy^2, 2xyz - yz^2) = (F_1, F_2, F_3)$$

What does this tell us about whether or not F is a gradient vector field?

7. (20 pts.) Find the line integral of the vector field

$F(x, y) = (x^2 - y^2, y^3 - 2xy)$

along the path $c(t) = (t, 1 - t)$, $0 \leq t \leq 1$.

9. (20 pts.) Find the area of the region D in the plane whose boundary is the parametrized curve

$c(t) = (4t - t^3, 2t - t^2)$, $0 \leq t \leq 2$ (see figure).

1. Find the velocity and acceleration of the parametric curve

$$(x(t), y(t)) = (t - \sin t, 1 + 2\cos t) \ .$$

6. Integrate the function $f(x, y, z) = z$ over the solid region R bounded by the x-y plane (i.e., $z = 0$) and the paraboloid

$$z = x^2 + y^2 - 9$$

(see figure). (Hint: a different coordinate system might simplify some of the calculation.)

7. Find the area of the region S bounded by one loop of the curve described by

$$r = \sin(3\theta)$$

in polar coordinates; see the figure below. (Hint: to determine the limits of integration, when is $r = 0$?)

8. Find the volume of the region T lying between the sphere $\rho = 3$ (in spherical coordinates) and the cone $\phi = \pi/6$; see figure below.