Math 208

What you should know and be able to do, for the final

11. Functions of several variables

Find the domain; sketch cross-sections, sketch contour diagrams/level curves.

12. Vectors

Go between $\vec{i}, \vec{j}, \vec{k}$ notation and coordinate notation; compute length, dot product, cross product; compute projection of one vector onto another; find equation of the plane passing through three points, find equation of a plane from a point and normal vector.

13. Differentiation

Compute partial derivatives, gradient, equation for tangent plane to the graph, higher order partial derivatives; compute the partial derivatives of a composite function using the Chain Rule for several variables. Remember that mixed partials are equal; gradient vectors are perpendicular to level curves.

14. Optimization

Find the critical points of a function of two or three variables; use the *Hessian* of f, $f_{xx}f_{yy} - (f_{xy})^2$, to distinguish between local max's, local mins, and saddle points; find the global max or min over a domain (unconstrained optimization); find the max or min for one function lying on the level curve of another (constrained optimization - Lagrange multipliers).

15. Integration

Understand that integrals are *sums*; compute the integral of a function of two variables over a region lying between two curves; know how to switch from $dy\ dx$ to $dx\ dy$; compute the triple integral over a region lying under a graph and over a region in the plane; determine the 'shadow' in the plane of a region in 3-space. Integrate a function over a cicular region using polar coordinates; integrate a function over a spherical region using cylindrical and spherical coordinates.

16. Parametrized curves

Sketch a curve from a parametrization; parametrize a circle and a line through a pair of points; compute the velocity, acceleration, and length of a parametrized curve; compute, from velocity and initial point, the parametrization of a curve.

17. Vector fields

Understand vector fields as a choice of vector at each point of a domain; sketch vector fields, e.g., gradient vector fields.

18. Line integrals

Undertand line integral as sum of dot products of $\vec{F}$ with velocity vectors; compute using a parametrization of a curve; the Fundamental Theorem of Line Integrals: the integral of a gradient field depends only of the endpoints; gradient fields are path independent.

Path independent vector fields are gradient fields; compute potential function by integrating coordinates. Compute the curl of a vector field; use Green's Theorem to compute a line integral over a closed curve (oriented correctly) as the integral of the curl over the region it bounds. Use to compute the area of a region by integrating a field with curl 1 around the boundary.

19. Flux integrals

Understand flux integral as the rate of flow of a fluid through a surface S whose velocity vectors are $\vec{F}$; understand as integrating $\vec{F} \bullet \vec{n}\ dA$ where $\vec{n}$ is the unit normal to the surface. Compute flux integrals in the special cases the S is a piece of the graph of a function ($\vec{n}\ dA = (-f_x, -f_y, 1)$, integrate over shadow of S in the plane), a piece of a cylinder (using cylindrical coordinate), or a piece of a sphere (using spherical coordinates).

20. Calculus of vector fields

Compute the divergence of a vector field, understand it as the flux density of the field; use the Divergence Theorem to compute a flux integral over the boundary of a region as a triple integral over the region; compute the flux integral of a divergence-free vector field as the flux integral of the field over another surface with the same boundary.

Compute the curl of a vector field, understand it as the circulation density of the field; use Stokes' Theorem to compute the line integral of $\vec{F}$ over a closed curve C as the flux integral of curl($\vec{F}$) over a surface with boundary C; compute the flux integral of a curl-free vector field over a surface S as the flux integral over another surface with the same boundary.