Math 208H

A checklist of topics covered

Parametric curves

sketch a curve from a parametrization
parametrize a circle, and a line through a pair of points
velocity, speed, acceleration
length of a parametrized curve, surface area of surface of revolution
integration: from velocity and initial point, find the parametrization of a curve.
area of the region enclosed by a parametric curve (see also *Green's Theorem*!)
polar coordinates; arclength and area formulas

Vectors

coordinate notation and $\vec{i}, \vec{j}, \vec{k}$ notation
length, dot product, cross product; orthogonal vectors
area via cross product; volume via triple product
projection of one vector onto another
equation of the plane passing through three points
equation of a plane from a point and normal vector.

Functions of several variables; differentiation

domain; sketch cross-sections, sketch contour diagrams/level curves.
partial derivatives, gradient, directional derivative
gradient vectors are perpendicular to level curves
equation for tangent plane to the graph
Chain Rule for several variables
higher order partial derivatives; mixed partials are equal
linear and quadratic approximations, differentials
differentiability
optimization; critical points
local max's, local mins, and saddle points via the discriminant $f_{xx}f_{yy} - (f_{xy})^2$
global max or min over a domain (unconstrained optimization)
optimization subject to a constraint - Lagrange multipliers

Integration

integrals are *sums*
integral of a function of two variables over a region in the plane
iterated integrals; reverse the order of integration
applications: area, mass, center of mass
change of variables formula; Jacobian
integrals in polar coordinates
surface area
triple integral over a region R in 3-space
iterated integrals; "shadow" of R in the plane
cylindrical and spherical coordinates.

Vector calculus

vector fields are a choice of vector at each point of a domain
sketch vector fields, e.g., gradient vector fields.
line integrals/path integrals
compute using a parametrization of a curve
the Fundamental Theorem of Line Integrals: integrating gradient fields
conservative vector fields; compute potential function
curl of a vector field
Green's Theorem
compute the area of a region via Green's Theorem
surface integrals/flux integrals
compute using a parametrization of a surface
special cases: graph of a function, cylinder, sphere

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divergence of a vector field
Divergence Theorem
volume of a region via Divergence Theorem
curl of a vector field
Stokes' Theorem