Math 208H, Section 1

Some (more) practice problems for the Final Exam

1. Find the length of the parametrized curve
$$\vec{r}(t) = (t^6 \cos t, t^6 \sin t) \quad , \quad 0 \le t \le \pi$$

2. Find the equation of the plane tangent to the graph of
$$z = f(x, y) = xe^y - \cos(2x + y)$$
at $(0, 0, -1)$
In what direction is this plane tilting up the most?

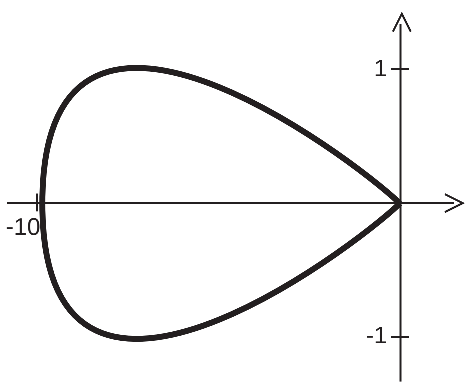
3. Find the critical points of the function
$$z = g(x, y) = x^2y^3 - 3y - 2x$$
and for each, determine if it is a local max, local min, or saddle point.

4. Find the integral of the function
$$z = h(x, y) = \ln(x^2 + y^2 + 1)$$
over the region
$$R = \{(x, y) : x^2 + y^2 \le 4\}$$

5. Find the integral of the function
$$k(x, y, z) = z$$
over the region lying inside of the sphere of radius 2 (centered at the origin $(0, 0, 0)$) and above the plane $z = 1$.

6. Show that the vector field $\vec{F} = \langle y^2, 2xy - 1\rangle$ is conservative, find a potential function $z = f(x, y)$ for $\vec{F}$, and use this potential function to (quickly!) find the integral of $\vec{F}$ along the path
$$\vec{r}(t) = (t\sin(2\pi t) - e^t, \ln(t^2 + 1) - 5t^2) \quad , \quad 0 \le t \le 1$$

7. Use Green's Theorem to find the area of the region enclosed by the curve
$$\vec{r}(t) = (t^2 - 2\pi t, \sin t) \quad , \quad 0 \le t \le 2\pi$$



8. Find the flux of the vector field
$\vec{G} = \langle x^2, xz, y\rangle$ through that part of the graph of

$$z = f(x,y) = xy$$

lying over the rectangle

$$1 \le x \le 3 \quad , \quad 0 \le y \le 3$$

1. Find the orthogonal projection of the vector $\vec{v}$ = (3,1,2) onto the vector $\vec{w}$ = (-1,4,2) .

2. Find the equation of the plane passing through the points

(1,1,1), (2,1,3), and (-1,2,1)

3. Use the tangent plane at (1,2,2) to approximate the value of

$$f(x,y) = (x)^{\frac{1}{2}}(4x+y^2)^{\frac{1}{3}}$$

for $(x,y) = (2,3)$

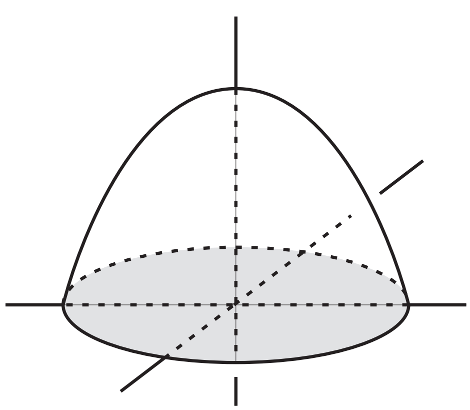
4. Find the integral of the function $f(x,y) = xy^2$
over the region in the plane lying between the graphs of
$a(x) = 2x$ and $b(x) = 3 - x^2$

5. Find the integral of the vector field $F(x,y) = (xy, x+y)$
along the parametrized curve $\vec{r}(t) = (e^t, e^{2t})$ $0 \le t \le 1)$.

6. Which of the following vector fields are **gradient** vector fields?

(a) $F(x,y) = (y\sin(xy), x\sin(xy))$
(b) $G(x,y,z) = (x^2y, z^2+x, 2yz)$
(c) $H(x,y,z) = (y+y^2z, x+2xyz, xy^2)$

7. Use the Divergence Theorem to find the flux integral of the vector field
$F(x,y,z) = (y, xy, z)$ through the boundary of the region lying under the graph of
$f(x,y) = 1 - x^2 - y^2$ and above the x-y plane (see figure).



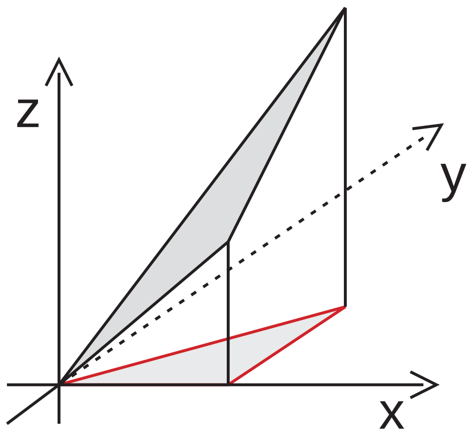
8. Use Stokes Theorem to find the line integral of the vector field

$$F(x, y, z) = (xy, xz, yz)$$

around the triangle with vertices

$$(0,0,0),\ (1,0,1),\ \text{and}\ (1,1,2)$$

(see figure).



9. Imagine a box with side lengths $x = 2$, $y = 3$, and $z = 4$, and these lengths all change with time. How fast is the volume of the box changing, if

$$\frac{dx}{dt} = 3,\ \frac{dy}{dt} = -2,\ \text{and}\ \frac{dz}{dt} = -1\ ?$$

10. Find the critical points of the function

$$f(x, y) = x^3y^2 - 6x^2 - y^2$$

and for each, determine if it is a rel max, rel min, or saddle point. Does the function have a global maximum?

11. By switching the order of integration, find the integral

$$\int_0^1 \int_x^1 xe^{\frac{x^2}{y}}\ dy\ dx$$

13. Find the flux integral of the vector field

$$F(x, y, z) = (1, y^2, xz)$$

over the sphere of radius 1 centered at (0,0,0) .