Name: Solutions

Math 221, Section 5

Quiz number 2

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find the (implicit) solutions to the differential equation

$$\frac{dy}{dx} = \frac{x\cos x}{\sqrt{y+1}}$$ separable!

and find the specific solution satisfying the additional initial value condition

$$y(0) = 8\,.$$

$$\sqrt{y+1}\,\frac{dy}{dx} = x\cos x$$

$$\int \sqrt{y+1}\,dy = \int \sqrt{y+1}\,\frac{dy}{dx}\,dx = \int x\cos x\,dx$$

$u = x \quad dv = \cos x\,dx$

$du = dx \quad v = \sin x$

$u = y+1$, $du = dy$

$$\int u^{1/2}\,du\Big|_{u=y+1} = x\sin x - \int \sin x\,dx$$

$$\frac{2}{3}u^{3/2}\Big|_{u=y+1} = x\sin x - (-\cos x) + C$$

$$\boxed{\frac{2}{3}(y+1)^{3/2} = x\sin x + \cos x + C}$$ implicit solutions

IVP:

$$\frac{2}{3}(8+1)^{3/2} = (0)(0) + (1) + C$$

$$C = \frac{2}{3}(9)^{3/2} - 1 = \frac{2}{3}(3)^3 - 1 = \frac{2}{3}(27) - 1 = 18 - 1 = 17.$$

$$\boxed{\frac{2}{3}(y+1)^{3/2} = x\sin x + \cos x + 17}$$ solution to IVP.

or (solve for y)

$$y = \left(\frac{3}{2}\left(x\sin x + \cos x + 17\right)\right)^{2/3} - 1$$