Math 221, Section 5

Quiz number 8 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find the general solution to the system of equations

$$x' = 3x - y$$
$$y' = 2x + y$$

Solving the first equation for y, we get (**) $y = 3x - x'$.

Substituting this into the second equation, we have

$(3x - x')' = 2x + (3x - x')$, so $3x' - x'' = 5x - x'$, so

$$(*)\ x'' - 4x' + 5x = 0$$

This is a second order homogeneous constant coefficients equation, which we can solve. The characteristic equation is

$$a^2 - 4a + 5 = 0$$

with solutions $a = \dfrac{4 \pm \sqrt{16 - 20}}{2} = \dfrac{4 \pm 2\iota}{2} = 2 \pm \iota$,

so the fundamental solutions to (*) are $x_1 = e^{2t}\cos t$ and $x_2 = e^{2t}\sin t$.

So the general solution is $x = c_1 e^{2t}\cos t + c_2 e^{2t}\sin t$.

Substituting this into (**), we get

$$\begin{aligned} y &= 3(c_1 e^{2t}\cos t + c_2 e^{2t}\sin t) - (c_1 e^{2t}\cos t + c_2 e^{2t}\sin t)' \\ &= [3c_1 e^{2t}\cos t + 3c_2 e^{2t}\sin t] - [c_1(2e^{2t}\cos t - e^{2t}\sin t) + c_2(2e^{2t}\sin t + e^{2t}\cos t)] \\ &= (e^{2t}\cos t)[3c_1 - 2c_1 - c_2] + (e^{2t}\sin t)[3c_2 + c_1 - 2c_2] \\ &= (c_1 - c_2)e^{2t}\cos t + (c_1 + c_2)e^{2t}\sin t\ . \end{aligned}$$

So the general solution to our system of differential equations is

$$x = c_1 e^{2t}\cos t + c_2 e^{2t}\sin t \qquad \text{and} \qquad y = (c_1 - c_2)e^{2t}\cos t + (c_1 + c_2)e^{2t}\sin t$$

for appropriate constants c_1 and c_2 (determined by a set of initial conditions).