Math 1650
Topics for first exam

Chapter 1: Functions and their graphs

§1: Graphs of equations
  Cartesian (x-y) plane
  graph = all points that *satisfy* the equation
  How to graph?
    plot points (and fill in gaps)
    use x- and y-intercepts
    use symmetry
      y-axis: (a,b) on graph, so is (-a,b)
      x-axis: (a,b) on graph, so is (a,-b)
      origin: (a,b) on graph, so is (-a,-b)
  Eqn for circle: $(x-h)^2+(y-k)^2=r^2$

§2: Lines and their slopes
  slope= rise over run = (change in y-value)/(corresponding change in x value)
  slope-intercept: $y=mx+b$
  point-slope: $\dfrac{y-y_0}{x-x_0}=m$
  two-point: $\dfrac{y-y_0}{x-x_0}=\dfrac{y_1-y_0}{x_1-x_0}$
  same slope: lines are parallel (do not meet)
  lines are perpendicular: slopes are **negative reciprocals**

§3: Functions
  function = rule which assigns to each input **exactly one** output
  inputs = domain; outputs = range/image; f:A→B
  y=f(x) : 'y equals f of x' : y equals the value assigned to x by the function f
  f,x,y, etc. are all placeholders; any other sybols are 'just as good'
  'implied' domain of f: all numbers for which f(x) *makes sense*

§4: Graphs of functions
  y=f(x) is an equation; graph the equation!
  graph = all pairs (x,f(x)) where x is in the domain of f
  all functions have graphs, but not all graphs 'have' functions
    function takes only one value at a point; vertical line test
  symmetry (for functions)
    y-axis: *even* function, f(-x) = f(x)
    x-axis: XXXXXX
    origin: *odd* function, f(-x) = -f(x)
  increasing on an interval: if $x>y$, then $f(x)>f(y)$
  decreasing on an interval: if $x>y$, then $f(x)<f(y)$
  constant

§5: Translations and combinations
  graph of y=f(x)
    shift to right by c; y=f(x-c)
    shift to left by c; y=f(x+c)
    shift down by c; y=f(x)-c
    shift up by c; y=f(x)+c
  y=*a*f(x) ; stretch graph by factor of *a*
  reflect graph along axes
    y-axis: y=f(-x)
    x-axis: y=-f(x)
  combining functions: combine the outputs of two functions f,g
    f+g, f-g, fg, f/g
  composition: output of one function is input of the next
  f followed by g = g∘f; g∘f(x) = g(f(x)) = g of f **of** x

§6: Inverse functions

Idea: find a function that undoes f

find a function g so that g(f(x)) = x for every x

magic: f undoes g ! Usual notation: g = f^{-1}

Problem: not every function has an inverse.

need g to be a function; so f cannot take the same value twice.

horizontal line test

Graph of inverse: if (a,b) on graph of f, then (b,a) is on graph of f^{-1}

graph of f^{-1} is graph of f, reflected across line y=x

Chapter 2: Polynomials

§1: Quadratic functions

monomial = ax^n

polynomial = bunch of monomials = $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = f(x)$

$a_n \neq 0$, then n=*degree* of f

deg=0: constant fcn; deg=1: linear fcn; deg=2: *quadratic* fcn

$f(x) = ax^2 + bx + c$; graph = *parabola*

Standard form: $ax^2 + bx + c = a(x-h)^2 + k$

complete the square: $ax^2 + bx + c = a(x^2 + \frac{b}{a}x) + c$

add half of $\frac{b}{a}$, squared, inside parentheses

(and subtract corresponding amount outside!)

standard form →graph:

x^2 to $(x-h)^2$ (shift left/right) to

$a(x-h)^2$ (stetch/reflect) to $a(x-h)^2 + k$ (shift up/down)

lowest/highest point of graph = (h,k) = *vertex* of parabola

axis of symmetry: vertical line x=h

§2: General properties of polynomials

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$; domain = everything

graph has no gaps, hole, or jumps (f is *continuous*)

can draw graph without lifting up writing implement

graph has no corners - no sudden turns; graph is *smooth*

behavior at 'ends':

n even, $a_n > 0$: high/high

n even, $a_n < 0$: low/low

n odd, $a_n > 0$: low/high

n odd, $a_n < 0$: high/low

root (zero) of f ; $f(a) = 0$; grpah of f hits x-axis at a

if $f(a) = 0$, then $f(x) = (x-a)g(x)$

nth degree polynomial can have at most n roots

nth degree polynomial can *turn around* at most (n-1) times

consequence of continuity: intermediate value theorem

if a polynomial takes on two values c and d, then

it also takes on every value in between

application: 'finding' roots: if $f(a) < 0$ and $f(b) > 0$, then

there is a root of f somewhere between a and b