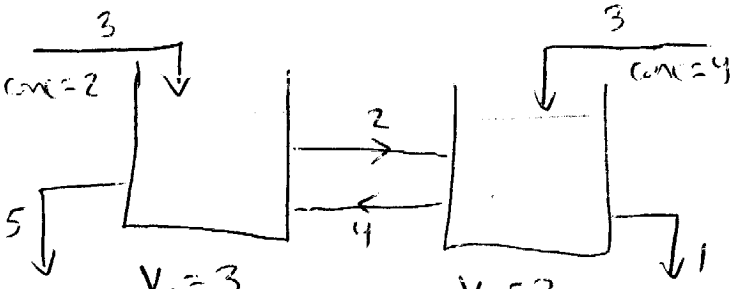
Math 221 Quiz 10 Solution



$V_x' = (3+4)-(5+2) = 0$

$V_x \equiv 3$

$V_y' = (2+3)-(4+1) = 0$

$V_y \equiv 2$

Tank x: $x' = \text{in} - \text{out}$

$= (3\cdot 2 + 4\cdot\frac{y}{2}) - (5\cdot\frac{x}{3} + 2\cdot\frac{x}{3})$

$= 6 + 2y - \frac{7}{3}x$

$y' = \text{in} - \text{out}$

$= (3\cdot 4 + 2\cdot\frac{x}{3}) - (4\cdot\frac{y}{2} + 1\cdot\frac{y}{2})$

$= 12 + \frac{2}{3}x - \frac{5}{2}y$

IVP:

$$x' = 6 + 2y - \tfrac{7}{3}x, \quad x(0)=3$$
$$y' = 12 + \tfrac{2}{3}x - \tfrac{5}{2}y, \quad y(0)=5$$

OR

$2y = x' + \frac{7}{3}x - 6$

$y = \frac{1}{2}(x' + \frac{7}{3}x - 6)$

$y' = \frac{1}{2}(x'' + \frac{7}{3}x')$

$\frac{1}{2}(x'' + \frac{7}{3}x') = 12 + \frac{2}{3}x - \frac{5}{2}\left(\frac{1}{2}(x' + \frac{7}{3}x - 6)\right)$

$\frac{1}{2}x'' + \frac{7}{6}x' = 12 + \frac{2}{3}x - \frac{5}{4}x' - \frac{35}{12}x + \frac{15}{2}$

$$\tfrac{1}{2}x'' + \left(\tfrac{7}{6}+\tfrac{5}{4}\right)x' + \left(\tfrac{35}{12} - \tfrac{2}{3}\right)x = \left(12 + \tfrac{15}{2}\right)$$

($\frac{29}{12}$, $\frac{9}{4}$, $\frac{39}{2}$)

$x(0) = 3$

$x'(0) = 6 + 2y(0) - \frac{7}{3}x(0)$

$= 6 + 2\cdot 5 - \frac{7}{3}\cdot 3$

$= 6 + 10 - 7 = 9$

$$x(0)=3, \quad x'(0)=9$$

$\frac{2}{3}x = y' + \frac{5}{2}y - 12$

$x = \frac{3}{2}(y' + \frac{5}{2}y - 12)$

$x' = \frac{3}{2}(y'' + \frac{5}{2}y')$

$\frac{3}{2}(y'' + \frac{5}{2}y') = 6 + 2y - \frac{7}{3}\left(\frac{3}{2}(y' + \frac{5}{2}y - 12)\right)$

$\frac{3}{2}y'' + \frac{15}{4}y' = 6 + 2y - \frac{7}{2}y' - \frac{35}{4}y + 42$

$$\tfrac{3}{2}y'' + \left(\tfrac{15}{4}+\tfrac{7}{2}\right)y' + \left(\tfrac{35}{4} - 2\right)y = 48$$

($\frac{29}{4}$, $\frac{27}{4}$)

$y(0) = 5$

$y'(0) = 12 + \frac{2}{3}x(0) - \frac{5}{2}y(0)$

$= 12 + \frac{2}{3}\cdot 3 - \frac{5}{2}\cdot 5$

$= 12 + 2 - \frac{25}{2} = 14 - \frac{25}{2} = \frac{3}{2}$

$$y(0)=5, \quad y'(0)=\tfrac{3}{2}$$