

Name: _____

Solution

Math 221, Section 3

Quiz number 2

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Use Euler's method, with a stepsize of $h = 1$, to estimate the value of the solution to the IVP

$$\frac{dy}{dt} = 1 - 2ty \quad , \quad y(0) = 2$$

at $t = 2$.

$$\begin{aligned} y_0 &= 2 \quad t_0 = 0 \quad m_0 = 1 - 2(0)(2) = 1 \\ y_1 &= 2 + 1 \cdot 1 = 3 \quad t_1 = 1 \quad m_1 = 1 - 2(1)(3) = -5 \\ y_2 &= 3 + (-5) \cdot 1 = -2 \quad t_2 = 2 \end{aligned}$$

$$y(2) \approx -2$$

2. Repeat problem 1, using a stepsize of $h = \frac{1}{2}$.

$$\begin{aligned} y_0 &= 2 \quad t_0 = 0 \quad m_0 = 1 - 2(0)(2) = 1 \\ y_1 &= 2 + 1 \cdot \frac{1}{2} = \frac{5}{2} \quad t_1 = \frac{1}{2} \quad m_1 = 1 - 2\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \\ &\quad = 1 - \frac{5}{2} = -\frac{3}{2} \\ y_2 &= \frac{5}{2} + \left(-\frac{3}{2}\right)\left(\frac{1}{2}\right) = \frac{5}{2} - \frac{3}{4} = \frac{7}{4} \\ t_2 &= \frac{1}{2} + \frac{1}{2} = 1 \quad m_2 = 1 - 2(1)\left(\frac{7}{4}\right) \\ &\quad = 1 - \frac{7}{2} = -\frac{5}{2} \\ y_3 &= \frac{7}{4} + \left(-\frac{5}{2}\right)\left(\frac{1}{2}\right) = \frac{7}{4} - \frac{5}{4} = \frac{1}{2} \\ t_3 &= \frac{1}{2} + 1 = \frac{3}{2} \quad m_3 = 1 - 2\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = 1 - \frac{3}{2} \\ &\quad = -\frac{1}{2} \\ y_4 &= \frac{1}{2} + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \quad t_4 = \frac{3}{2} + \frac{1}{2} = 2 \\ y(2) &\approx \frac{1}{4} \end{aligned}$$