Name: Solutions

Math 221, Section 3

Quiz number 6

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1 (a): Show that $y = \sin x$ is a solution to the differential equation

$$y'' + (2\tan x)y' - y = 0$$

$y = \sin x$
$y' = \cos x$
$y'' = -\sin x$

$$y'' + (2\tan x)y' - y = -\sin x + \left(2\frac{\sin x}{\cos x}\right)\cos x - \sin x$$
$$= -\sin x + 2\sin x - \sin x = 0 \checkmark$$

1 (b): Use reduction of order to find a second, linearly independent, solution to the differential equation.

$y = c(x)\cdot\sin x$ $\qquad$ $c(x) = \int \frac{e^{-\int 2\tan x\,dx}}{(\sin x)^2}\,dx$

$\int \tan x = \ln|\sec x|$ $\qquad$ $e^{-\int 2\tan x\,dx} = e^{-2\ln(\sec x)}$

$= (\sec x)^{-2} = \cos^2 x$

$$c(x) = \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \cot^2 x\,dx = \int \csc^2 x - 1\,dx$$
$$= -\cot x - x$$

So $y = c(x)\sin x = \left(-\frac{\cos x}{\sin x} - x\right)\sin x = -\cos x - x\sin x$

is a second solution.

Check: $y = -\cos x - x\sin x$
$y' = \sin x - \sin x - x\cos x$
$= -x\cos x$
$y'' = -\cos x + x\sin x$

$y'' + 2\frac{\sin x}{\cos x}y' - y$
$= -\cos x + x\sin x + (-2x\sin x) + \cos x + x\sin x$
$= (1 - 2 + 1)x\sin x = 0. \checkmark$