Solution

Name:

Math 221, Section 3

Quiz number 8

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Find the general solution to the differential equation

$$y'' + y = \sin x + 2x - 1$$

homog. eqn $y''+y=0$ $r^2+1=0$ $r=\pm i$

homog solns: $y=\cos x$ $y=\sin x$

$y''+y=\sin x$ Use $y = Ax\cos x + Bx\sin x$

$y' = A\cos x - Ax\sin x + B\sin x + Bx\cos x$

$y'' = -A\sin x - A\sin x - Ax\cos x + B\cos x + B\cos x - Bx\sin x$

$= -2A\sin x - Ax\cos x + 2B\cos x - Bx\sin x$

$y''+y = -2A\sin x - 2B\cos x = \sin x \rightsquigarrow -2A=1,\ -2B=0$

$A=-\frac{1}{2}$, $B=0$ $y = -\frac{1}{2}x\cos x$

$y''+y = 2x-1$ Use $y=ax+b$ $y'=a$, $y''=0$

$y''+y = (0)+(ax+b) = 2x-1$

$a=2$, $b=-1$ $y = 2x-1$ (!)

So general solution is

$$y = c_1\cos x + c_2\sin x - \frac{1}{2}x\cos x + 2x - 1$$

$y'' + y = \sin x \qquad y = c_1 \cos x + c_2 \sin x$ | By Variation of Parameters...

$$c_1 = \int \frac{\begin{vmatrix} 0 & \sin x \\ \sin x & \cos x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} dx = \int -\sin^2 x \, dx = \int \frac{1}{2}(\cos 2x - 1)\, dx$$

$$= \frac{1}{4}\sin 2x - \frac{1}{2}x$$

$$c_2 = \int \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \sin x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} dx = \int \sin x \cos x \, dx = \int \frac{1}{2}\sin 2x \, dx$$

$$= -\frac{1}{4}\cos 2x$$

So $y = \left(\frac{1}{4}\sin 2x - \frac{1}{2}x\right)\cos x - \left(\frac{1}{4}\cos 2x\right)\sin x$

$= (\text{trig identities!}) = \frac{1}{4}\sin x - \frac{1}{2}x\cos x$.

$y'' + y = 2x - 1 \qquad y = c_1 \cos x + c_2 \sin x$

$$c_1 = \int \frac{\begin{vmatrix} 0 & \sin x \\ 2x-1 & \cos x \end{vmatrix}}{1} = \int \sin x - 2x\sin x \, dx$$

($v = -\cos x$, $du = 2dx$)

$$= -\cos x - \left[-2x\cos x + \int 2\cos x\right] = -\cos x + 2x\cos x - 2\sin x$$

$$c_2 = \int \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & 2x-1 \end{vmatrix}}{1} dx = \int 2x\cos x - \cos x \, dx$$

($v = \sin x$, $du = 2dx$)

$$= -\sin x + 2\left[x\sin x - \int \sin x \, dx\right] = -\sin x + 2x\sin x + 2\cos x$$

So $y = (-\cos x + 2x\cos x - 2\sin x)\cos x + (-\sin x + 2x\sin x + 2\cos x)\sin x$

$= (\text{trig identities!}) = 2x - 1$. ∎