

### Math 310 Homework 3

Due Tuesday, September 25

13. Use the Euclidean Algorithm to find the gcd of  $a = 1111$  and  $b = 473$ , then reverse the steps of your calculations to write  $\gcd(a, b)$  as a combination of  $a$  and  $b$ .
14. Repeat problem 13, with the numbers  $a = 1357$  and  $b = 2468$ .
15. (Childs, p.49, E1) Show by induction that if a prime number  $p$  divides the product  $a_1 \cdots a_n$  of  $n$  integers  $a_1, \dots, a_n$ , then  $p$  divides at least one of the  $a_i$ .
16. Show that if  $a$  is an integer,  $n \geq 1$ ,  $p$  is prime, and  $p|a^n$ , then  $p^n|a^n$ .
17. (Childs, p.50, E3) Show that if  $n \geq 1$  is *not* prime, then  $n$  can be factored as  $n = pq$  where  $p$  is prime and  $p \leq \sqrt{n}$ . Use this to determine whether or not 239 is prime.

**For Math 310H, or extra credit:**

- H2. (Childs, p.51, E5) Show that if  $a$  and  $b$  are integers, both  $\geq 1$  and with  $(a, b) = 1$ , and  $ab = c^r$ , then  $a = x^r$  and  $b = y^r$  for some integers  $x$  and  $y$ .  
(Hint: Ignore Childs' hint, he was trying to be too clever. One approach is to use complete induction (on  $c$ ).)