Math 310 Homework 4

Due Tuesday, October 2

18. (Childs, p.67, E2 (sort of)) Show that if a is an *odd* integer and $ab \equiv ac \pmod 8$, then $b \equiv c \pmod 8$.

    What are some other numbers besides 8 for which this will work?

19. Show by induction (on n) that if the integers $a_1, \ldots, a_n$ are all congruent to 1 modulo m, then their product $a_1 \cdot \cdots \cdot a_n \equiv 1 \pmod m$.

20. (Childs, p.67, E4) Show that if $a \equiv b \pmod m$ and n is any natural number, then $a^n \equiv b^n \pmod m$.

21. (Childs, p67, E6) Find the remainder of each of the numbers a when you divide by the corresponding number b :

    (1): $a = 5^{18}$, $b = 7$

    (2): $a = 68^{105}$, $b = 13$

    (3): $a = 6^{47}$, $b = 12$

    (Hint: Problem 20, suitably applied, will help!)

22. Show that if a and b are integers with $a \equiv b \pmod p$ for *every* prime p, then $a = b$.

    (Hint: How big is $|b - a|$?)

For Math 310H, or extra credit:

H3. Show that every integer of the form $n = 4m + 3$ has a prime factor of the form $4k + 3$. (Translation: every number that leaves remainder 3 upon division by 4 has a prime factor that leaves remainder 3 upon division by 4.) Use this to show that there are infinitely many prime numbers of the form $4m + 3$.

Hint (for both parts!): What is the alternative? Mimic our proof of the infinitude (what a wonderful word....) of primes, for the second part....