

## Math 310 Homework 8

Due Tuesday, November 13

38. Show that if  $R \cong S$  and  $R$  is an integral domain, then so is  $S$ .
39. If  $R$  is a ring with  $0_R \neq 1_R$ , then an element  $a \in R$  cannot be *both* a zero divisor *and* a unit.
40. Show that the ring  $\mathbb{Z}_5[i] = \{a + bi : a, b \in \mathbb{Z}_5\}$ , with addition and multiplication defined as in problem 36, is *not* a field.
41. Show that “is isomorphic to” is an equivalence relation, i.e, for any three rings  $R$ ,  $S$ , and  $T$ ,
- (a)  $R \cong R$
  - (b) If  $R \cong S$ , then  $S \cong R$
  - (c) If  $R \cong S$  and  $S \cong T$ , then  $R \cong T$
- (Hint: the “obvious” functions work, but don’t forget to show that each is both bijective *and* a homomorphism!)

### For Math 310H, or extra credit:

- H5. Let  $n$  be a positive integer that is *not* the square of another integer (so that  $\sqrt{n}$  is not rational). Let

$$\mathbb{Q}[\sqrt{n}] = \{a + b\sqrt{n} : a, b \in \mathbb{Q}\} \subseteq \mathbb{R}$$

with the usual addition and multiplication from  $\mathbb{R}$ . Show that  $\mathbb{Q}[\sqrt{n}]$  is a *subfield* of  $\mathbb{R}$ , i.e., it is both a subring and a field in its own right.