Math 314 Exam 1 Practice Problems

Show all work. Include all steps necessary to arrive at an answer unaided by a mechanical computational device. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

Note: The problems below include no resource allocation, balancing chemical equation, or network flow problems. Such problems may appear on our exam!

1. (20 pts.) Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{rcrcrcl} -x & & & + & 2z & = & 3 \\ x & - & y & - & 3z & = & 3 \\ 2x & + & 3y & - & 3z & = & 6 \end{array}$$

2. (20 pts.) Use row reduction to decide if the system of linear equations given by the augmented matrix:

$$(A|\mathbf{b}) = \left(\begin{array}{cccc|c} -1 & 0 & 2 & 3 & 2 \\ 3 & 2 & -4 & 1 & -2 \\ 0 & 3 & 0 & 1 & 6 \end{array}\right)$$

has a solution. If it does, does it have one or more than one solution?

3. (25 pts.) Is the vector $\vec{b} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$ in the span of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$?

More generally, what (linear) equation among a, b, c must hold in order for $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to be in the span of $\vec{v}_1, \vec{v}_2$?

4. (20 pts.) Use Gauss-Jordan elimination to find the inverse of the matrix A, where

$$\text{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 5 & 8 \end{pmatrix}$$

(b) (5 pts.) Use your answer from to find the solution to the equation $\text{A}x = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

5. (10 pts.) Let $\mathbf{O}$ denote the $n \times n$ matrix with all entries equal to 0.

Suppose that A and B are $n \times n$ matrices with

$$AB = \mathbf{O} \ ,$$

but

$$B \neq \mathbf{O} \ .$$

Show that A **cannot** be invertible.
(Hint: suppose it *is*: what does that tell you about B ?)

1. (20 pts.) Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{ccccccc} 4x & + & 4y & + & 2z & = & 3 \\ x & + & 3y & + & 2z & = & 1 \\ 3x & + & 2y & + & z & = & 1 \end{array}$$

2. (20 pts.) A pet hotel can accept 50 dogs and 70 cats in its care. The average Belgian owns 2 dogs and 1 cat, while the average Luxembourgian owns 1 dog and 3 cats. How many Belgians and Luxembourgians can the pet hotel accomodate, if all of the available space is used?

3. (25 pts.) Show that the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

are linearly dependent, and exhibit an explicit linear dependence among them.

4. (25 pts.) Use row reduction to find the values of x for which the following matrix is **invertible**:

$$\text{A(x)} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & x & -1 \\ 2 & 2 & -x \end{pmatrix}$$

5. (10 pts.) An $n \times n$ matrix A is called **nilpotent** if $A^k = 0_{n\times n}$ for some number k. Show that if A is nilpotent, then $I - A$ is an **invertible** matrix. [Hint: "factor" $I = I - A^k = (I - A)(\text{what?})$. You can try $k = 2, 3$, or 4 first to give you some feel for the general case...]