Math 314/814, Section 4

Quiz number 1 Solution

Find a solution to the system of equations

$$\begin{aligned} 2x - y + 2z &= 3 \\ x - y + 3z &= -1 \\ -2x + 5y + z &= -2 \end{aligned}$$

Solution: There are any number of ways to solve this. Here is one.

Rewiting this in matrix form, and applying row reduction steps:

Start: $\begin{pmatrix} 2 & -1 & 2 & | & 3 \\ 1 & -1 & 3 & | & -1 \\ -2 & 5 & 1 & | & -2 \end{pmatrix}$

swap rows: $\begin{pmatrix} 1 & -1 & 3 & | & -1 \\ 2 & -1 & 2 & | & 3 \\ -2 & 5 & 1 & | & -2 \end{pmatrix}$

add multiples of top row: $\begin{pmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & -4 & | & 5 \\ -2 & 5 & 1 & | & -2 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & -4 & | & 5 \\ 0 & 3 & 7 & | & -4 \end{pmatrix}$

add multiple of middle row: $\begin{pmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & -4 & | & 5 \\ 0 & 0 & 19 & | & -19 \end{pmatrix}$

rescale bottom row: $\begin{pmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & -4 & | & 5 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$

Then we can either backsolve: $z = -1$; $y - 4z = y + 4 = 5$, so $y = 1$;
and $x - y + 3z = x - (1) + (-3) = x - 4 = -1$, so $x = 3$, or continue row reduction:

add multiples of bottom row: $\begin{pmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 0 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$

add multiple of middle row: $\begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$

so $x = 3$, $y = 1$, $z = -1$.

So $x = 3$, $y = 1$, $z = -1$ is a solution to our original system of equations. (It is also the only solution...)

Note that we can check that our answer is right, by plugging our solution values into the system:

$2(3) - (1) + 2(-1) = 6 - 1 - 2 = 3$ $\qquad$ $(3) - (1) + 3(-1) = 3 - 1 - 3 = -1$

$$-2(3) + 5(1) + (-1) = -6 + 5 - 1 = -2$$