Math 314/814, Section 4

Quiz number 2 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Show that the vector $\begin{bmatrix}3\\1\\2\\3\end{bmatrix}$ is in the span of the vectors $\begin{bmatrix}1\\3\\4\\1\end{bmatrix}$, $\begin{bmatrix}4\\2\\3\\3\end{bmatrix}$, and $\begin{bmatrix}1\\1\\2\\2\end{bmatrix}$.

(Express it as a linear combination.)

Solution: This translates to finding a solution to the linear system $\begin{pmatrix}1&4&1&|&3\\3&2&1&|&1\\4&3&2&|&2\\1&3&2&|&3\end{pmatrix}$.

We proceed by row reduction:

$$\begin{pmatrix}1&4&1&|&3\\3&2&1&|&1\\4&3&2&|&2\\1&3&2&|&3\end{pmatrix} \to \begin{pmatrix}1&4&1&|&3\\0&-10&-2&|&-8\\0&-13&-2&|&-10\\0&-1&1&|&0\end{pmatrix} \to \begin{pmatrix}1&4&1&|&3\\0&-1&1&|&0\\0&-10&-2&|&-8\\0&-13&-2&|&-10\end{pmatrix}$$

$$\to \begin{pmatrix}1&4&1&|&3\\0&1&-1&|&0\\0&-10&-2&|&-8\\0&-13&-2&|&-10\end{pmatrix} \to \begin{pmatrix}1&4&1&|&3\\0&1&-1&|&0\\0&0&-12&|&-8\\0&0&-15&|&-10\end{pmatrix} \to$$

$$\begin{pmatrix}1&4&1&|&3\\0&1&-1&|&0\\0&0&1&|&2/3\\0&0&-15&|&-10\end{pmatrix} \to \begin{pmatrix}1&4&1&|&3\\0&1&-1&|&0\\0&0&1&|&2/3\\0&0&0&|&0\end{pmatrix} \to \begin{pmatrix}1&4&1&|&3\\0&1&-1&|&0\\0&0&1&|&2/3\\0&0&0&|&0\end{pmatrix}$$

$$\to \begin{pmatrix}1&4&0&|&7/3\\0&1&0&|&2/3\\0&0&1&|&2/3\\0&0&0&|&0\end{pmatrix} \to \begin{pmatrix}1&0&0&|&-1/3\\0&1&0&|&2/3\\0&0&1&|&2/3\\0&0&0&|&0\end{pmatrix}$$

So the system is consistent, and there is a solution. In particular,

$$\begin{bmatrix}3\\1\\2\\3\end{bmatrix} = -\frac{1}{3}\begin{bmatrix}1\\3\\4\\1\end{bmatrix} + \frac{2}{3}\begin{bmatrix}4\\2\\3\\3\end{bmatrix} + \frac{2}{3}\begin{bmatrix}1\\1\\2\\2\end{bmatrix}$$