

Quiz number 3 solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

In one model of an economy, there are four commodities, A, B, C, D , which each serve as resources for the productions of the others. The production of A requires 40% of the resources for each of B, C, D . Production of B requires no D , and uses 20% of A and C . Production of C requires no A , and uses 10% of B and D . Production of D requires 20% of A , and 10% of each of B and C . Finally, the remaining fraction of each commodity is used as a resource for that commodity.

Based on this information, find the system of equations that will determine the equilibrium prices A, B, C, D of each commodity, so that the revenues generated equal the cost of the resources for each commodity, and express this system as an augmented matrix. **Do not solve this system** (unless you are really bored...).

B, C , and D use a total of $.2A + 0A + .2A = .4A$, so A uses $.6A$.

A, C , and D use a total of $.4B + .1B + .1B = .6B$, so B uses $.4B$.

A, B , and D use a total of $.4C + .2C + .1C = .7C$, so C uses $.3C$.

A, B , and C use a total of $.4D + 0D + .1D = .5D$, so D uses $.5D$.

Therefore, to balance revenue with cost, we need:

$$\begin{array}{ll} A = .6A + .4B + .4C + .4D, & \text{so } -.4A + .4B + .4C + .4D = 0; \\ B = .2A + .4B + .2C + 0D, & \text{so } .2A - .6B + .2C = 0; \\ C = 0A + .1B + .3C + .1D, & \text{so } .1B - .7C + .1D = 0; \\ D = .2A + .1B + .1C + .5D, & \text{so } .2A + .1B + .1C - .5D = 0. \end{array}$$

This gives the system of equations:

$$\begin{array}{l} -.4A + .4B + .4C + .4D = 0; \\ .2A - .6B + .2C = 0; \\ .1B - .7C + .1D = 0; \\ .2A + .1B + .1C - .5D = 0; \end{array}$$

Writing this as an augmented matrix, we have

$$\left(\begin{array}{cccc|c} -.4 & .4 & .4 & .4 & 0 \\ .2 & -.6 & .2 & 0 & 0 \\ 0 & .1 & -.7 & .1 & 0 \\ .2 & .1 & .1 & -.5 & 0 \end{array} \right)$$